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COMPENDIUM ON WATERWAY TRANSPORTATION RELIABILITY: LOCK CONGESTION AND LOCK QUEUES

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COMPENDIUM ON WATERWAY TRANSPORTATION RELIABILITY: LOCK CONGESTION AND LOCK QUEUES

by

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Water Resources Support Center
Institute for Water Resources
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April 1993

IWR Report 93-R-9

FOREWORD

The papers presented in this Compendium report research results on problems and proposed solutions on lock congestion and lock queues. This research was done under separate Corps of Engineers contracts with Dr. Paul Schonfeld, Department of Civil Engineering, and Dr. Harry Kelejian, Department of Economics, University of Maryland, College Park, MD.

The Editorial Board of the Transportation Research Record gave special permission for several of the articles to be reproduced in this Compendium. We are appreciative of Ms. Nancy Ackerman, Director of Reports and Editorial Services for the National Research Council, Transportation Research Board (TRB), Washington, D.C. for her support.

The research results present suggestions or alternative solutions for improving the reliability of the inland waterways transportation system. We believe the research provides a stimulus for useful discussion at the Division and District levels. Any comments on information presented in the articles should be addressed to the Navigation Division of the Institute for Water Resources.

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SIMULATION OF WATERWAY TRANSPORTATION RELIABILITY

by

Melody D. M. Dai and Paul Schonfeld

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Simulation of Waterway Transportation Reliability

MELODY D. M. DAI AND PAUL SCHONFELD

A microscopic model for simulating barge traffic through a series of locks has been developed and tested with data for a section of the Ohio River. The model was designed primarily to analyze the economic effects of waterway congestion and service reliability. The results indicate that the model is capable of simulating the system performance sufficiently well for analytic purposes. The results also indicate to what extent coal stockouts would increase at a power plant, or alternatively, how safety stocks would have to be increased, as traffic volumes approach capacity.

The reliability of service times on inland waterways significantly influences barge fleet requirements, operating costs, inventory costs, and stock out costs for customers. Therefore, the service reliability influences the competitive position and market share of inland waterway transportation.

To analyze the effects of congestion and service time variability, a simulation model has been developed. In its earliest applications for which results are presented, the model is used to estimate the relations among capacity and service time variance at successive locks, stock-out probabilities and durations, and inventory safety stocks for an electric power plant supplied with coal through the Ohio River. This model will soon be usable for estimating the benefits and costs of alternative plans for maintaining and improving the waterway system.

LITERATURE REVIEW

The research most relevant here regards the economic costs of lock delays, lock delay models, and waterway simulation models. The estimation of economic benefits is essential for selecting and scheduling lock improvement projects. The U.S. Army Corps of Engineers, which is the agency responsible for U.S. waterways, usually estimates the economic benefits of lock improvements from the transport cost differentials between barges and the next cheapest mode (*1–3*). Such evaluation omits some important logistics costs (e.g., for larger inventories and barge fleet sizes) used to hedge against unreliable deliveries.

In systems with unreliable deliveries, stockouts may occur. There are situations in which the on-site stocks are not sufficient to satisfy the demand (*4*). Stock-out costs include duplicate ordering costs from another source or mode and foregone profits (*5,6*). Baumol and Vinod (*7*) indicate that delays can increase the shippers' inventory costs which include on-site carrying costs and stock-out penalties. On the basis of Baumol's model, Nason and Kullman (*8*) developed a total logistics cost model to predict inland diversions from waterways.

Two models based on queueing theory have been found for estimating lock delays. DeSalvo (*9*) models lock operation as a simple single-server queueing process with Poisson distributed arrivals and exponentially distributed service times (i.e., M/M/1 queues). Wilson's model (*10*) extends DeSalvo's by treating the service processes as general distributions (M/G/1 queues). Both models are designed for analyzing single lock delays. However, the assumption of exponentially distributed service times is not consistent with empirical data (*11*) and the Poisson arrivals assumption is also unreliable. Carroll and Desai (*12,13*) studied the arrival processes at 40 locks on the Illinois, Mississippi, and Ohio river systems, and found that 13 of the 40 locks had non-Poisson arrivals at the five percent significance level.

The results for M/M/1 queues in DeSalvo's model (*9*) are derived on the basis of first-in-first-out (FIFO) service discipline although the actual discipline is primarily one-up-one-down. This assumption can still generate reasonable results since delays mainly depend on volume to capacity ratios. Wilson (*10*) modeled the service processes more realistically with a general rather than an exponential distribution. However, arrivals are still assumed to be Poisson distributed at all locks and no exact queueing results are available for locks with two chambers in parallel. Since analytic queueing models must be kept simple to be solvable, the above two models also neglect the interdependence among serial locks and the stalls (i.e., service interruptions at locks). Both of these factors significantly affect service times and reliability.

The system simulation models developed to analyze lock delays and two travel times originated mainly from Howe's microscopic model (*14*). In that model service times are derived from empirically determined frequency distributions. To avoid some troublesome problems and errors associated with the requirement to balance long-run flows in Howe's model, Carroll and Bronzini developed another waterway system simulation model (*15*). It provides detailed outputs on such variables as two traffic volumes, delays, processing times, transit times, averages and standard deviations of delay and transit times, queue lengths, and lock utilization ratios. Both of these models simulate waterway operations in detail but require considerable amounts of data and computer time, which limit their applicability for problems with large networks with numerous combinations of improvement alternatives. Both models assume a Poisson distribution for two trip generations, which is not always realistic. More important for reliability analyses, neither of these models explicitly accounts for stalls, which

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re different in frequency and duration from other events and iave significant effects on overall transit time reliability.

Hence a waterway simulation model that explicitly accounts or stalls and estimates the effects of service unreliability of nventory costs is desirable for evaluating and scheduling lock mprovement projects.

IMULATION MODEL

urpose

A waterway simulation model was developed to analyze the relations between tow trips, travel times, delays, lock operations, coal consumption, and coal inventories while taking account of stochastic effects and seasonal variations. This simulation model enables the estimation of inventory levels and expected stock-out amounts for coal, tow travel times along he waterway, and tow delays under a variety of assumptions about tow trip generation, tow motion, lock service, lock operation discipline, coal inventory level, and coal consumption. These estimates are useful for estimating economic benefits of lock improvements.

Features

This simulation model is focused on how variations in lock service times affect tow delays and how variations in tow delays affect coal inventories. The output of this model can provide the necessary information to estimate inventory costs, stock-out costs, and expected benefits resulting from lock rehabilitation or lock construction.

This simulation model is microscopic. It traces the motion and records the characteristics of each tow. The characteristics of tows include their number of barges, commodity types, speed, origin and destination, direction of motion, and arrival time at various points. In addition, the model determines cumulative deliveries, cumulative consumption, and actual inventories at various plants.

This is an event-scanning simulation model—the status of which is updated by events. There are five types of events. One is the generation of tow trips, which are generated stochastically on the basis of actually observed traffic distributions. The model uses a table to represent the trip generation pattern and is, therefore, not limited to standard mathematical probability distributions.

A second type of event is the tow entrance in a lock, which is determined by tow arrival time at that lock, the times when chambers become available, and the chamber assignment discipline. If a tow arrives before the lock is available, it needs to wait in the queue storage area. Otherwise, it is served according to the chamber assignment discipline, discussed later. In general, the lock service is presently "first come first serve," subject to the chamber assignment procedure.

A third type of event is a coal tow's arrival at its destination, which increases the cumulative deliveries by the amount of coal that tow is carrying. The cumulative consumption and inventory at the destination are also updated then.

A fourth type of event is the update in the status of cumulative consumptions, inventories, and consumption rates for all coal destinations every unit time. This provides detailed information on inventory levels for all coal destinations.

A fifth type of event is a lock stall. Whenever a stall occurs, the affected chamber becomes unavailable until the end of the stall.

The size of problem that the model can handle is limited by the computer capacity and the storage capacity of the Fortran compiler or linker. There are no restrictions on the number of locks, chambers, cuts, waterway links, tows, utility plants, origin-destination (O-D) pairs, and simulation time periods. This model can simulate two way operation on a mainline waterway.

This model is programmed in Fortran-77, which allows the simulation of relatively complex operations. The following is a more detailed description of how tow trip generation, tow travel times, and coal inventory levels are computed. The overall structure of the simulation model is displayed in Figure 1.

Tow Trip Generation

Tow trips are generated randomly, but the mean of their generating distribution is constant for each O-D pair over each simulation time period. The distribution for tow trip generation is represented by a table. It is assumed that the distribution of trip generation times is similar to the distribution of trip arrival times to locks, (for which data are available).

This model assumes that each tow will maintain its size through its trip. As in trip generation, tow sizes (numbers of barges per tow) are also generated randomly. The distribution of tow sizes is represented by a table and is assumed to be the same for each O-D pair. The tow size table is determined from input data and can represent tow size distribution.

Tow traffic is divided into coal and non-coal traffic. Therefore, for the same O-D pairs, there may be different trip rates

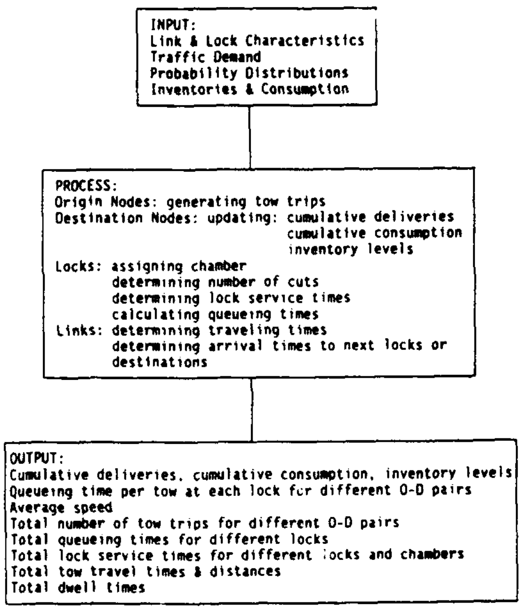

FIGURE 1 Structure and elements of the simulation model.

for coal and non-coal traffic. When coal tows arrive at their destinations, the model updates inventory levels. It is assumed that only a specified fraction of the barges on a coal tow are carrying coal.

Tow Travel Times

Tow travel times are estimated separately for each waterway section, queue storage area, and lock. Section travel times between locks and/or piers are determined by speeds and distances to be covered. Tow speeds are specified as an input to the model in the form of a probability distribution. The distribution of speeds is assumed to be normal. The model assumes that tows maintain constant speeds between origins and destinations and that backhaul speeds are a constant ratio of linehaul speeds.

To avoid generating extreme speed values, a speed range is specified. If speeds are lower than the 2.5 percentile speed or zero, or higher than the 97.5 percentile speed, the speeds are regenerated.

Queueing times at locks are a major focus of this simulation model. Such queueing delays may occur well before traffic levels approach lock capacity since tow arrivals and lock service times are not uniform. These delay times are computed from the difference between the tow arrival times at the queue storage area and their departure from the queue to enter the lock. The storage area has unlimited capacity and is adjacent to the lock.

Lock service times are generated from a specified distribution table. The distribution table can directly reflect actually observed service times. Therefore, the model can be applied to any type of locks. Lock service times will be affected by lock improvements, which are represented by smaller average lock service times or reduced service time distributions. The average lock service times vary for differentl locks, chambers, and numbers of cuts.

The number of cuts is determined by chamber and tow sizes. The maximum cut size (barges handled simultaneously) is exogenously specified for each chamber. A tow may be divided into different numbers of cuts at different lock chambers.

If a lock has more than one chamber in parallel, (main and auxiliary chambers are usually provided), it is currently assumed that the main chamber will be preferred, unless the additional wait time it requires (compared to the auxiliary chamber) exceeds a specified level. This lock selection bias factor reflects the additional work and delays required to break tows into more (and smaller) cuts, move them separately through the auxiliary chamber and then reassemble them. This bias factor has been estimated separately for various locks from empirical data.

The lock service discipline is currently "first come first serve." It is expected that the "N up-N down" service discipline will be simulated later.

Stalls

Stalls are failure conditions in which chambers are not available to serve tows. Stall characteristics differ among chambers and are defined in terms of durations and frequencies, which depend on weather conditions and lock conditions at each chamber. The model assumes that stalls occur stochastically with an exponential distribution.

Inventory Levels

Inventory levels are represented by the difference between cumulative deliveries and cumulative consumption. Whenever inventory levels drop to negative values, this model computes stock-out amounts and durations for the analysis of total costs. This model updates cumulative deliveries and cumulative consumption whenever coal tows arrive at destinations.

Cumulative deliveries are determined from initial inventory levels, inter-delivery times, and delivery amounts. The initial inventory level is exogenously specified for each destination (utility plant). The interdelivery time is generated by the simulation model. The delivered amount is determined from the barge payload and the number of arriving coal barges. The barge payload is currently assumed to be constant for each tow. The number of coal barges is currently assumed to be a constant fraction of tow size. The coal barge fractions vary for different O-D pairs. Although coal barge fractions are constant throughout the simulated period, the amount delivered by each tow is not constant since tow sizes are randomly generated.

Cumulative consumption is a function of consumption rate and time. The mean consumption rate is constant for each utility plant during each simulation period, although it fluctuates randomly around its mean. However, a constant rate is assumed within each period. The consumption rate is updated every time unit and is, therefore, a step-wise linear distribution over time, whose slopes are consumption rates.

Input Requirements

Generally, the model requires four types of inputs related to (*a*) link and lock characteristics, (*b*) traffic demand between origins and destinations, (*c*) probability distributions, and (*d*) inventories and consumption.

Link and Lock Characteristics

The following kinds of information are needed for each link: (*a*) end nodes, (*b*) link length, (*c*) distances between the end nodes and the lock, (*d*) number of chambers, (*e*) average frequencies and durations of stalls, (*f*) maximum cut sizes of chambers, (*g*) average service times of chambers for cuts of various sizes, (*h*) maximum number of barges for each cut size at each chamber, (*i*) bias time for each auxiliary chamber, and (*j*) random number seeds.

Traffic Demand

Traffic demand in tows per day is expressed in the form of O-D matrices by time periods. The lengths of time periods may be different and need to be specified. Additional information needed includes (*a*) dwell time at origins and destinations (both average and standard deviation); (*b*) average number of barges per tow for each O-D pair; (*c*) fractions of coal barges in a tow for each O-D pair; (*d*) payload in short-tons; (*e*) speed (both average and standard deviation); and (*f*) ratio of backhaul speed to linehaul speed (empty/full or upstream/downstream).

Probability Distributions

Probability distributions are specified in this model for (*a*) lock service timers, (*b*) trip generation, (*c*) tow composition (barges per tow), and (*d*) coal consumption at power plants.

The probability distribution tables represent cumulative distribution curves, wherein the abscissas are cumulative frequency, and the ordinates represent the ratio of the tabulated variable to its mean. To reduce the input complexity and specify only ordinates, a specified number of equal intervals is currently used for any cumulative frequency distribution.

Inventories and Consumption

Initial inventory levels in short-tons for different nodes (utility plants) must be specified. In addition, consumption rates in short-tons per day are expressed in the form of node matrices by time period. The information on cumulative deliveries, cumulative consumption, and inventory levels, is provided for intervals whose duration in days must be specified.

Model Output

This model prints out the following results: (*a*) total tow travel time (not including the queueing time, lockage time, and dwell time) in days; (*b*) total tow travel distances in 1,000 mi; (*c*) total dwell times at origins and destinations in days; (*d*) total queueing times in days for different locks and chambers; (*e*) total lock service times in days for different locks, chambers and cuts; (*f*) total number of tow trips for different O-D pairs; (*g*) average speed in mi per day; (*h*) queueing time (both average and standard deviation) in days per tow at each lock for different O-D pairs; (*i*) monthly cumulative deliveries, cumulative consumption, and inventory levels tables in 1,000 short-tons for different utility plants; (*j*) cumulative deliveries, cumulative consumption, and inventory levels tables for specified intervals in 1,000 short-tons for different utility companies; (*k*) graphs of cumulative deliveries and cumulative consumption by specified time intervals for different utility plants; and (*l*) graphs of inventory level by specified time interval for different utility plants.

CASE STUDY

A five-lock section of the Ohio River, centered on the Gallipolis Lock was selected for a case study because that lock constitutes a relative bottleneck in the water capacity. Compared with the four locks nearest to it, (Belleville, Racine, Greenup, and Meldahl), Gallipolis is the oldest and its two chambers are the smallest. A new Gallipolis lock chamber is under construction. The physical characteristics of these six locks are given in Table 1.

In general, a new lock will provide better service quality by reducing service time and improving reliability. The prior expectation is that electric utility plants served by a waterway may be able to reduce the required inventory levels and the expected stock-out costs if the service reliability on the waterway is improved.

The objective of this case study is to compare the inventory levels and expected stock-out amounts of a utility plant downstream of Gallipolis for cases with and without a new Gallipolis lock.

The Stuart utility plant, which belongs to Dayton Power and Light Co., was chosen for this case study. It is located between the Greenup and Meldahl locks. It is 63.5 mi downstream from Greenup and 31.7 mi upstream from Meldahl.

Model Application

This case study focuses on the Ohio river between the Belleville and Greenup locks. Although the model can simulate multiple plants, only one utility plant was analyzed. It included O-D pairs. The simulation period is 1 year.

Link and Lock Characteristics

To simulate the operation between Belleville and Greenup, five nodes and four links are used. The link characteristics are shown in Table 2. The lock characteristics are shown in Table 3.

It is noted that except for Node 5, which represents the Stuart utility plant, all nodes are null nodes that are used as the origins and destinations of non-coal traffic to generate equivalent volumes and congestion levels.

For existing locks, the average lock service times are determined according to the 1984 lock data. Because the new Gallipolis Lock is still under construction, its service times were not available and had to be estimated. The estimated values are slightly smaller than those of the four older locks, which have similar chamber sizes, because the newer lock is assumed to improve service.

TABLE 1 PHYSICAL CHARACTERISTICS OF LOCKS

	Chambers			
Lock Name	Year Opened	Width (ft)	Length (ft)	Lift (ft)
Belleville	1968	110	1200	22
	1968	110	600	22
Racine	1971	110	1200	22
	1971	110	600	22
Gallipolis	1937	110	600	23
	1937	110	360	23
Gallipolis	1991	110	1200	23
(new)	1991	110	600	23
Greenup	1959	110	1200	30
	1959	110	600	30
Meldahl	1962	110	1200	30
	1962	110	600	30

TABLE 2 LINK CHARACTERISTICS

Link	Lock Name	Node In	Node Out	Length (mi)	Distance Between In Node & lock (mi)
1	Belleville	1	2	37.9	21.1
2	Racine	2	3	37.6	16.8
3	Gallipolis	3	4	51.8	20.9
4	Greenup	4	5	94.4	30.9

TABLE 3 LOCK CHARACTERISTICS

Lock Name	Average Service time (in days per cut) 1 cut	2 cuts	Upper Limit of Cut size (in barges per cut)
Belleville	.03512	.09823	18*
	.02389	.07682	8*
Racine	.03425	.09579	18*
	.02427	.07805	8*
Gallipolis	.03563	.07840	6*
	.02088	.06173	3*
Gallipolis (new)	.03000	.09000	18**
	.01600	.07000	9**
Greenup	.03267	.09213	18*
	.02027	.08108	8*

* : based on PMS data
**: based on chamber dimensions

Traffic Demand and Consumption

There were five O-D pairs in this case study. O-D Pair 1 represents coal traffic for the Stuart plant. The other five O-D pairs are non-coal traffic or coal traffic for other utility plants.

The baseline values for average trip rates and tow sizes are determined from 1984 data, and are shown in Tables 4 and 5. The average consumption rates over 12 mo for the Stuart power plant were determined from 1984 coal consumption data and are shown in Table 6.

Other Parameters

The mean and standard deviation of downstream tow speeds are 9.02 and 2.82 mph (216.48 and 67.68 mi/day), respectively. The ratio of upstream speeds to downstream speeds is 0.83. These values were developed on the basis of 1983 statistical data of vessel performance on inland waterways. The barge payload was assumed to be 1,400 long tons or 1568 short tons. (One long ton = 2,240 lbs whereas a short ton = 2,000 lbs.)

Model Validation

The ability of the model to realistically simulate actual operating conditions may be assessed by comparing predictions with actual data. Tables 7 through 9 show such comparisons between results of simulation runs of one year and actual data from 1984. Table 7 shows that traffic volumes are predicted quite accurately by the model, with an average deviation of 1.53 percent. Table 8 shows that the waiting time in queues

TABLE 4 AVERAGE TRIP RATES

Month	Trip Rate (tows/day) O-D pair 1-5	1-2	2-3	3-4	4-5
Jan.	1.98	3.06	3.23	3.36	4.88
Feb.	1.97	3.48	3.46	3.76	5.65
Mar.	1.36	4.43	4.77	4.43	6.06
Apr.	1.57	4.68	4.63	3.74	6.31
May	2.27	4.31	4.39	4.33	5.52
June	1.91	6.31	5.76	5.91	10.44
July	2.20	5.55	5.20	5.15	9.10
Aug.	1.98	5.63	5.84	5.54	8.36
Sep.	2.08	5.07	5.61	5.42	8.71
Oct.	2.40	2.44	3.24	3.37	6.55
Nov.	2.13	2.43	2.92	2.95	5.82
Dec.	1.22	2.77	3.14	3.96	6.30

TABLE 5 AVERAGE TOW SIZES

O-D Pair	Tow Size (barges/tow)
1-5	6.8
1-2	9.1
2-3	9.4
3-4	8.4
4-5	6.7

TABLE 6 AVERAGE CONSUMPTION RATES AT THE STUART POWER PLANT

Month	Consumption Rate (1000 short-tons/day)
Jan.	17.23
Feb.	18.03
Mar.	15.26
Apr.	14.90
May	18.35
June	16.70
July	17.32
Aug.	18.52
Sep.	18.80
Oct.	16.29
Nov.	15.33
Dec.	16.48

TABLE 7 TRAFFIC VOLUME COMPARISON

Lock	Volume (tows/year) Data	Model	Deviation (%)
Belleville	4466	4292	3.90
Racine	4591	4580	0.24
Gallipolis	4575	4622	1.03
Greenup	6511	6450	0.94

TABLE 8 WAITING TIME COMPARISON

Lock	Wait Time(min/tow) Data	Wait Time(min/tow) Model	Deviation (%)
Belleville	21.45	21.81	1.68
Racine	17.26	15.22	11.82
Gallipolis	200.53	137.33	31.52
Greenup	14.46	13.31	7.95

TABLE 9 RELATIVE UTILIZATION OF LOCK CHAMBERS (VOLUMES ARE GIVEN IN TOWS/YEAR)

Lock	Data Main Chamber	Data Total Lock	Data %Main	Model Main Chamber	Model Total Lock	Model %Main	Deviation (%)
Belleville	3332	4466	74.61	3134	4292	73.02	2.13
Racine	3848	4591	83.82	3851	4580	84.08	0.31
Gallipolis	3656	4575	79.91	3488	4622	75.47	5.56
Greenup	4500	6511	69.11	4891	6450	75.83	9.72

is predicted reasonably well by the model, although the model significantly underestimates the delays at the Gallipolis Lock. That lock has unusual operating characteristics because it requires disassembly of tows into exceptionally small and oddly composed cuts. A more detailed analysis of operations at Gallipolis may be required to more accurately model its peculiarities. Table 9 shows that the model can satisfactorily estimate the relative utilization of the two chambers at each lock, with an average deviation of 4.43 percent. It should be noted that the model predictions are not only close to actual observation, but are also not systematically biased in any particular direction.

System Congestion and Reliability

In waterways, as in other transportation systems, delays increase much faster than volumes as the capacity is approached and tend toward infinite values. Moreover, the relative variance of service times (e.g., the coefficient of variation = standard deviation divided by the mean) is expected to increase faster than the average service times, with unfavorable effects on system reliability. In a linear network such as that in our case study, the capacity of the entire system is limited by the capacity of the most constrictive element in the series, namely the Gallipolis Lock. Because a new lock will be opened in 1991, which will match the capacity of Gallipolis to that of the other locks in the series, we present simulation results for both the old and new locks.

Table 10 shows the effects of traffic volumes and safety stocks on expected stock-out amounts. It is evident that as volumes (both coal traffic and non-coal traffic) increase from baseline levels (1.0) to levels 50 and 100 percent higher (i.e., volume ratios of 1.5 and 2.0, respectively), the stock-out amounts increase more than proportionately. As safety stock levels are increased from 0 to 150,000 and 300,000 tons, the stock-out amounts consistently decrease. The rate of decrease tapers off (to zero, eventually) as safety stocks are increased.

The effect on stock outs of the new higher capacity Gallipolis Lock is nearly negligible at current volumes (volume ratio = 1.0). However as volumes double, its effect becomes quite significant, since the old lock would reach a utilization rate of 82.85 percent (i.e., 83 percent of capacity). In this case, the decrease in stock-outs ranges from 60,850 tons/day (= 363,010 − 302,160) or 16.76 percent at zero safety stock to 54,790 tons/day or 40.73 percent at a safety stock of 300,000 tons.

Table 11 shows the effects on stock outs of stalls (failures) at locks. The stalls column indicates stall frequency. Thus 1 indicates baseline conditions (i.e., frequency based on 1980–1987 data), whereas 2 and 3 indicate that frequency is doubled and tripled, respectively. The predicted stock-out amounts are g[illegible] for both the old and new Gallipolis Locks in the format [illegible]w. The results show that stall duration and frequencies have relatively slight effects on stock outs when volumes are low, that is, when comparing Case 2 or Case 4 with the baseline Case 1. However at high volumes (Cases 9–12), when the system operates closer to its capacity, the effects of stalls become significant and the advantage of the higher capacity of the new Gallipolis Lock is quite substantial.

Total System Costs

The results of this work show how expected stock-out levels increase disproportionately with congestion levels (i.e., volume to capacity ratios) and decrease (with diminishing returns) as safety stocks are increased. Figure 2 shows how the total system costs depend on holding costs and stock-out costs. Holding costs, which include storage costs and interest charges on the safety stock are indicated by the linear function H in the Figure 2. The holding cost is assumed to be $0.10/ton-

TABLE 10 EXPECTED STOCK-OUT AMOUNTS FOR VARIOUS SAFETY STOCK LEVELS AND VOLUMES

Gallipolis Lock	Volume Ratio	Utilization of Gallipolis lock %	Expected Stock-Out Amount (1000 short-tons/day) Safety Stock(1000 short-tons) 0	150	300
Old	1.0	38.19	220.88	91.41	7.28
	1.5	59.19	258.58	125.33	29.23
	2.0	82.85	363.01	236.17	134.52
New	1.0	18.73	219.74	90.50	6.97
	1.5	27.42	254.04	121.26	26.73
	2.0	35.92	302.16	176.38	79.73

TABLE 11 EXPECTED STOCK-OUT AMOUNTS (IN 1,000 TONS/DAY)

Case	Multiplier: Volume	Multiplier: Stalls	Multiplier: Stall Duration	Starting Inventory (1000 tons): 0	150	300
1	1	1	1	220.1/219.7	91.41/90.50	7.28/6.97
2	1	1	2	221.1/220.0	91.56/90.71	7.29/7.01
3	1	1	3	222.4/220.5	97.69/91.29	7.54/7.10
4	1	2	1	221.2/220.0	91.66/90.73	7.37/7.00
5	1	3	1	221.5/220.2	91.86/90.89	7.49/7.13
6	1.5	1	1	257.2/254.0	124.1/121.3	28.30/26.73
7	1.5	2	1	259.1/254.6	125.9/121.8	29.14/26.89
8	1.5	3	1	261.6/255.8	128.3/122.9	30.89/27.64
9	2	1	1	363.0/302.2	236.2/176.4	134.5/79.73
10	2	2	1	416.4/302.8	282.0/177.0	174.9/83.10
11	2	3	1	579.6/306.2	435.8/180.0	311.7/82.54
12	2	3	2	823.1/320.8	678.5/193.7	553.0/92.80

Key: Expected stock-out amounts given with OLD/NEW Gallipolis Lock. Multipliers are ratios of ASSUMED/BASELINE values. Case 1 represents baseline values.

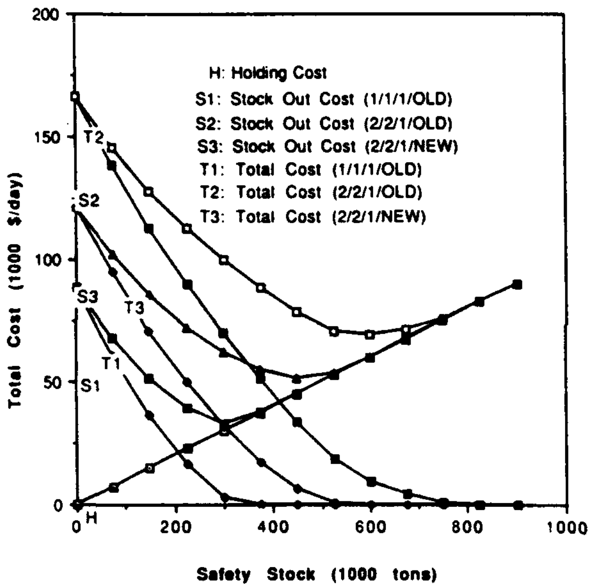

FIGURE 2 Effect of holding costs and stock-out costs on total system costs.

day. If that holding cost were doubled, the slope of the function H would double.

Figure 2 shows the stock out costs for three combinations of parameters, using the key VOLUME/STALL FREQUENCY/STALL DURATION/GALLIPOLIS LOCK. Thus, according to this key, 2/2/1/OLD means that volumes and stall frequencies are twice the baseline values, stall durations are equal to baseline values, and the old Gallipolis lock is being simulated. It should be remembered that our baseline volumes represent 1984 data. A cost of $0.40/ton is assumed in computing the stock out cost curves of Figure 2.

The total system cost is obtained by adding the holding cost to the stock out costs. Because the holding cost is the same for all cases in Figure 2, we obtain one total system cost function for each of the three stock out cost functions. The total cost curves show that as volumes and stall frequencies double (from 1/1/1/OLD to 2/2/1/OLD) the optimal safety stock levels should approximately double from 300,000 to 600,000 tons and that total system costs would more than double from approximately $33,000/day to $69,000/day. If, however, the new Gallipolis Lock was operational, the optimal safety stock level would only be approximately 450,000 tons and the system cost would be approximately $51,000/day, despite doubled volumes and stall frequencies. The curves in Figure 2 show quite clearly the tradeoffs between increased safety locks and increased stock out costs.

Figures 3 through 5 repeat the analysis of Figure 2 with various assumptions about the cost of holding safety stock and the cost of stocking out. They show that as stock out costs increase relative to holding costs, the optimal amounts of safety stocks should increase.

It should be noted that the only sources of delivery unreliability modeled so far are lock operations and lock failures. Safety stock policies of utilities might also be affected by other factors such as probabilistic expectations of coal mine strikes, frozen waterways and coal price changes. It is possible that such factors may dominate the effects of lock performance analyzed to date.

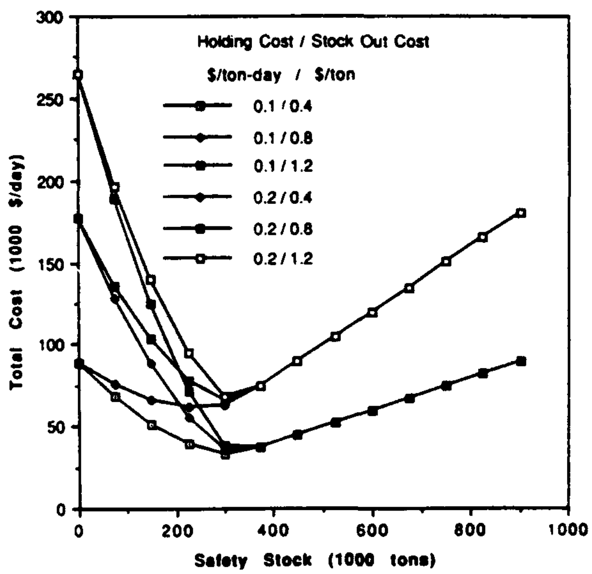

FIGURE 3 Total system costs (1/1/1/OLD).

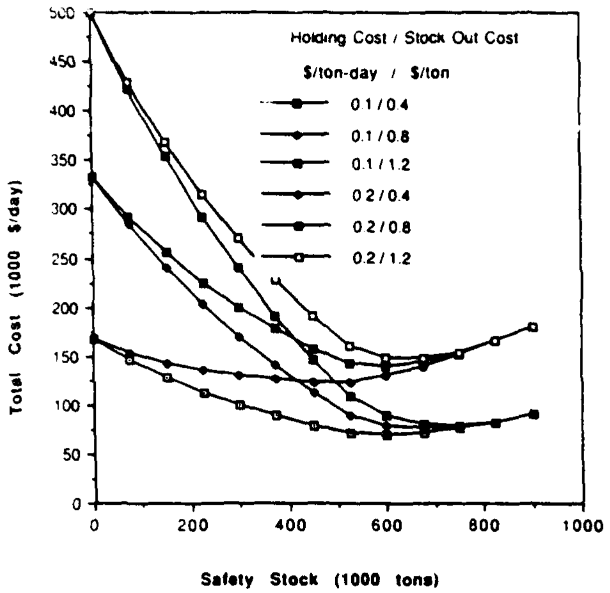

FIGURE 4 Total system costs (2/2/1/OLD).

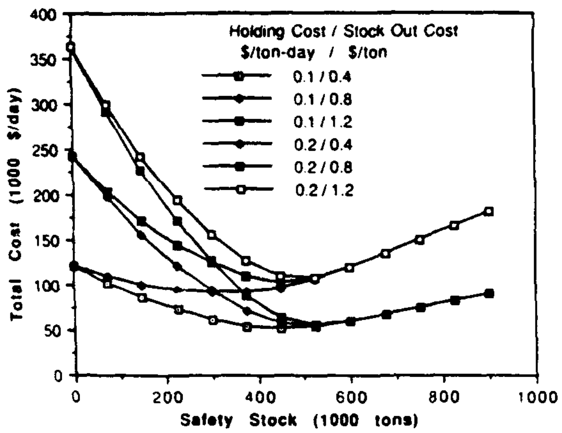

FIGURE 5 Total system costs (2/2/1/NEW).

CONCLUSIONS

Waterway Congestion and Reliability

The results of this work show how expected stock-out levels increase disproportionately with congestion levels (i.e., volume to capacity ratios) and decrese (with diminishing returns) as safety stocks are increased. Such results provide the basis for tradeoffs between inventory holding costs and stock-out costs. The optimized safety stocks resulting from such tradeoffs, and hence their holding costs, would increase as congestion increases and transit time reliability decreases in the system. Such effects are relatively slight when volume to capacity ratios are small. If and when volumes increase substantially above present levels, reliability benefits can justify capacity improvements such as the new Gallipolis Lock.

Model Capability

The simulation model provides estimates of system performance that are sufficiently detailed and accurate for analytic purposes, although its computer requirements are quite modest for a microscopic simulation model. The model's accuracy might be improved by improvements in traffic generation, tow composition, lock selection, and failure generation functions. These improvements might be developed on the basis of a more extensive analysis of empirical data and, possibly, on lock maintenance and failure research. The model may also be extended to translate physical performance measures such as fleet requirements, delays, safety stocks, and stock outs into monetary costs and benefits. Finally, more macroscopic versions of the model are being developed to efficiently analyze alternative investment and maintenance strategies for the national waterway system.

ACKNOWLEDGMENTS

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PROBABILITY MODEL OF LOCKAGE STALLS AND INTERFERENCES

by

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Probability Model of Lockage Stalls and Interferences

HARRY H. KELEJIAN

A model of lock failures as manifested by stalls or interferences and specified in terms of a logit formulation is presented in this paper. Stalls or interferences that correspond to commercial tow and recreational vessel lockages and that result from lock hardware problems or to testing or maintaining the lock or its equipment are first considered. The expected frequency of such lock failures relative to the number of commercial tow and recreational vessel lockages is then explained. These expected frequencies can be viewed as measures of reliability or interpreted as the probability that such failures will occur on any given commercial tow or recreational vessel lockage. The qualitative results corresponding to the underlying variables are consistent with expectations. The usefulness and flexibility of the model in evaluating changes in the values of these variables is demonstrated. Among other things, this demonstration suggests that many major maintenance projects relating to lock chambers can be evaluated by their consequent effect on lock failure probabilities. It is demonstrated that the extent of the renewal of a chamber in response to major maintenance can be calculated.

The following scenario was suggested in a recent study (Charles Yoe, unpublished data). The Army Corps of Engineers operates and maintains 260 lock chambers and 536 dams at 596 sites. These structures are in various states of repair, performance, and obsolescence. Many of them are older than their original 50-year design life. Maintenance, repair, major maintenance, and replacement of these facilities are becoming increasingly necessary and increasingly costly. Furthermore, the recent inland navigation investment program, as reflected by total appropriations for general construction and operations and maintenance has declined from $689 million in fiscal year 1980 to $655 million in fiscal year 1987. After adjusting for price level differences, this 5 percent nominal decline becomes a 35 percent real decline. Continued and even increasing strain on fiscal resources is expected for the foreseeable future. Further details are given elsewhere (*1*).

As a result of increasing needs and decreasing fiscal resources to meet those needs the Corps' decision problem is how best to allocate scarce resources to operation, maintenance, repair, major maintenance, and replacement of structures on the inland waterway. In evaluating the economic impacts of many of these investment decisions, it is necessary to quantify the costs of increasingly unreliable or insufficient service at locks and/or the benefits of improving reliability or increasing capacity. This analysis of reliability generally requires an effort to quantify the probabilities of impaired lock services with and without proposed projects.

This study presents a model of lock failures as manifested by stalls or interferences. A stall is an occurrence which stops lock operation. An interference is an occurrence which slows lock operation during a lockage. For more detail see the Corps' *User's Manual for Data Analysis* (*2*).

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The model considers stalls or interferences (henceforth, stalls) that correspond to commercial tow and recreational vessel lockages and that result from lock hardware problems or to testing or maintaining the lock or its equipment. It then explains the expected frequency of such lock failures relative to the number of commercial tow and recreational vessel lockages. This can be viewed as a measure of reliability. It can also be interpreted as the probability that such a failure will occur on any given commercial tow or recreational vessel lockage.

The model is specified in terms of a logit formulation. The explanatory variables relate to characteristics of the lock chambers, to the extent of major maintenance (if any), and to variables which identify the Corps of Engineer district the lock chamber is associated with. Among other things, the usefulness of the model as a tool of prediction and as an instrument for allocating major maintenance funds is demonstrated.

DATA ISSUES

The data underlying this study were taken from the U.S. Army Corps of Engineers' Lock Performance Monitoring System (PMS) data tapes, details of which are reported elsewhere (*2*). The data taken from these tapes relate to lockages at 125 lock chambers for 1981 through 1986. These lock chambers correspond to 14 Corps of Engineer districts. The 125 lock chambers were chosen from the entire list of lock chambers described in the PMS tapes because the corresponding data were of a higher quality in the sense that fewer errors were present and more complete in terms of having fewer missing observations. Data relating to an individual lockage at these 125 chambers were not used unless observations on all of the relevant variables were available.

A description of the 125 lock chambers is contained in Kelejian (*3*). The 14 districts corresponding to these 125 lock chambers are listed in Table 1. It became convenient to describe each district by a number (i.e., District 1, District 2, etc.). These district numbers are also listed in Table 1.

The data file used to estimate the model contained two types of PMS data. The first relates to individual lockages. The second relates to calendar year sums (e.g., total stall time). The individual lockage data represents a one-out-of-twelve sample from the original PMS data tape. The annual sums are based on a 100 percent sample.

TABLE 1 DISTRICTS AND THEIR ASSOCIATED NUMBERS

District	Associated Number	District	Associated Number
Pittsburgh	1	Huntington	8
Mobile	2	St. Louis	9
Nashville	3	St. Paul	10
Walla Walla	4	Little Rock	11
Wilmington	5	Tulsa	12
Louisville	6	Vicksburg	13
Rock Island	7	Seattle	14

The data file also contained information pertinent to lock chambers described in (*1*). Among other things, this information relates to the age of lock chambers and the cost of completed major maintenance projects. The cost of the maintenance projects were given in current dollars. These data were converted into constant 1982 dollars by deflating by the Construction Cost Index. Data on this index were supplied by Corps personnel. A more complete discussion of the data and their original source is given in Kelejian (*3*).

LOGIT MODEL OF STALL PROBABILITY

Basic Formulation: An Overview

There are typically many lockages that take place during a year at a given lock chamber. Corresponding to each of these lockages there is a probability that a stall will occur.

Let P_{it} be the probability that a commercial tow or recreational vessel lockage taking place during the tth year at lock chamber i results in a stall. Note that P_{it} is indexed to vary from lock chamber to lock chamber (over i) and from year to year, but not from one lockage to another within a year at a given chamber.

The assumption that the probability of a stall is the same for all lockages taking place within a year at a given chamber is clearly an approximation. For example, a lock chamber ages continuously and, therefore, from lockage to lockage. However, one might view the effective aging of a chamber as being very gradual and therefore reasonably well approximated by the age of the chamber as measured in years. If so, and if the other relevant factors change gradually from lockage to lockage, the assumption of a constant stall probability within a year at a given chamber is reasonable.

Let X_{it} be a vector of variables corresponding to the ith lock at time t, which might be taken to explain P_{it}. Let B be a corresponding vector of parameters such that

$$I_{it} = X_{it}B \quad (1)$$

can be taken to be an index determining P_{it}. Then, in the logit formulation P_{it} is related to I_{it} as

$$P_{it} = EXP(I_{it})/[1 + EXP(I_{it})] \quad (2)$$

It is not difficult to show that P_{it} lies between zero and unity for all possible values of the index I_{it}. In addition $(dP_{it}/dI_{it}) > 0$ for all I_{it} so that the larger is the index I_{it} the higher is P_{it}. Therefore, variables that are components of I_{it} that increase P_{it} should have positive weights; negative weights correspond to variables which decrease P_{it}.

Details of the Index

In this study, the index relating to the ith chamber at time t, namely I_{it}, is

$$\begin{aligned} I_{it} = {} & b_0 + b_1\,\mathrm{Age}_{it} + b_2\,\mathrm{MPT}_{it-1} + b_3\,\mathrm{ICE}_{it-1} \\ & + b_4\,\mathrm{AIT}_{it} + b_5\,\mathrm{Maint}_{it} + b_6\,\mathrm{ST}_{it-1} \\ & + b_7\,\mathrm{SF}_{it-1} + a_1\,\mathrm{DD1}_i + a_2\,\mathrm{DD3}_i \\ & + a_3\,\mathrm{DD7}_i + a_4\,\mathrm{DD8}_i + a_5\,\mathrm{DD10}_i \end{aligned} \quad (3)$$

where $b_0, \ldots, b_7, a_1, \ldots a_5$ are parameters to be estimated and all of the remaining terms on the right hand side of expression 3 are explanatory variables whose definitions are given in Table 2.

In expression 3 Maint_{it} represents the extent of a major maintenance, if any. It was formulated as

$$\mathrm{Maint}_{it} = 1 - \mathrm{EXP}(-\mathrm{cost}_{it}) \quad (4)$$

where $\mathrm{cost}_{it} = 0$ if, up through time t, lock chamber i did not have major maintenance; if such maintenance did take place, cost_{it} is its 1982 dollar cost. The specification in expression 4 implies that $\mathrm{Maint}_{it} = 0$ if $\mathrm{cost}_{it} = 0$. This is the case in which a major maintenance did not take place. If it did take place, $\mathrm{cost}_{it} > 0$ and so $\mathrm{Maint}_{it} > 0$, and the more extensive it was (the higher is cost_{it}), the higher is Maint_{it}. In this sense, the variable Maint_{it} is a positive measure of the extent of a major maintenance.

A number of other variables were also considered but found not to be statistically significant. Results relating to these other variables can be found elsewhere (*3*).

Since age, other things being equal, is associated with lock deterioration, one would expect $b_1 > 0$. Similarly, higher values of mean processing time may be indicative of equipment which is not in top operating condition and so one ex-

TABLE 2 DEFINITIONS OF EXPLANATORY VARIABLES

Variable	Definition
Age_{it}	The age of lock chamber i at time t
MPT_{it-1}	Mean processing time of lock chamber i at time t-1.
ICE_{it-1}	The number of ice days at lock chamber i during year t -1.
AIT_{it}	Average idle time at lock chamber i during year t.
Maint_{it}	A variable describing the real dollar value of a major maintenance (if any) of lock chamber i.
ST_{it-1}	Total stall time due to testing or maintenance of lock chamber i, or its equipment during year t-1.
SF_{it-1}	The stall frequency at lock chamber i at year t-1. The stall frequency is the ratio of stalls to lockages.
DDJ_i	A dummy variable which is unity if the ith lock chamber is in District J, and zero otherwise.

pects $b_2 > 0$. One would also expect $b_3 > 0$ and $b_7 > 0$ because ice formation accelerates decay and a previous stall frequency is indicated of general conditions, which are not radically different from one year to the next.

One would expect $b_4 < 0$. The reason for this is that idle time could be used to perform minor maintenance and repair, and so on. Thus, higher values of AIT_{it} should lower the index, I_{it}, and hence lower the probability of a stall. Similarly, for very evident reasons one expects $b_5 < 0$. On a somewhat more moderated scale, one would also expect $b_6 < 0$. That is, the more testing and maintenance, and corresponding minor repairs, of the lock chamber and its equipment in one year, the better the condition (other things equal) of that chamber in the following year. For the readers' convenience, the sign expectations relating to the coefficients of expression 3 are summarized in expression 5 as follows:

$$b_1 > 0,\ b_2 > 0,\ b_3 > 0,\ b_7 > 0;$$
$$b_4 < 0,\ b_5 < 0,\ b_6 < 0 \qquad (5)$$

The coefficient of a dummy variable in expression 3 indicates whether or not the stall probability corresponding to that district is higher (if positive) or lower (if negative) than in the districts not represented in expression 3 after the effects of the other variables in the index have been accounted for. Conceptual arguments do not suggest the signs of these coefficients.

The Issue of Estimation

Assume that P_{it} is neither zero nor unity. Then from expression 2 it can be shown that $P_{it}/(1 - P_{it}) = \mathrm{EXP}(I_{it})$ so that

$$\log_e (P_{it}/(1 - P_{it})) = I_{it} \qquad (6)$$

The result in expression 6 is useful in that it leads to a relatively simple procedure for estimating the parameters determining the index I_{it} as given in expression 3. For example, let SF_{it} be the number of stalls at lock chamber i during year t. Then SF_{it} may be expressed as $SF_{it} = S_{it}/L_{it}$ where S_{it} is the number of stalls of the type being considered at lock chamber i during year t, and L_{it} is the corresponding number of lockages. Because the probability of a stall on any given lockage is assumed to be the same for all lockages during the year at a given chamber, SF_{it} can be taken as an estimate of P_{it}. The reason for this is that SF_{it} can be viewed as the ratio of the number of successes (stalls) to the number of trials (lockages).

For ease of presentation, suppose that SF_{it} is neither zero nor unity. Then let

$$u_{it} = \log_e[SF_{it}/(1 - SF_{it})] - \log_e(P_{it}/(1 - P_{it})) \qquad (7)$$

so that

$$\log_e(SF_{it}/(1 - SF_{it})) = \log_e[P_{it}/(1 - P_{it})] + u_{it} \qquad (8)$$

The first term on the right hand side of expression 8 is equal to the index I_{it} via expression 6. Replacing this index by its expression in expression 3 yields

$$\begin{aligned}\log_e(SF_{it}/(1 - SF_{it})) = {} & b_0 + b_1\,\mathrm{Age}_{it} + b_2\,\mathrm{MPT}_{it-1} \\ & + b_3\,\mathrm{ICE}_{it-1} + b_4\,\mathrm{AIT}_{it} \\ & + b_5\,\mathrm{Maint}_{it} + b_6\,\mathrm{ST}_{it-1} \\ & + b_7\,\mathrm{SF}_{it-1} + a_1\,\mathrm{DD1}_i \\ & + a_2\,\mathrm{DD3}_i + a_3\,\mathrm{DD7}_i \\ & + a_4\,\mathrm{DD8}_i + a_5\,\mathrm{DD10}_i \\ & + \mathrm{uit} \end{aligned} \qquad (9)$$

It can be shown that if the number of lockages during year t at chamber i is "large", the term u_{it} has a mean and variance which are approximated by the following expression

$$Eu_{it} = 0,\ \mathrm{Var}(u_{it}) = [L_{it}\, P_{it}\, (1 - P_{it})]^{-1} \qquad (10)$$

The implication of expression 10 is that expression 9 can be viewed as a regression model having a heteroskedastic error term. Because the variance of u_{it} involves P_{it}, which is not known, the appropriate estimation procedure is a feasible form of generalized least squares that is based on an estimated value of the variance of u_{it}; this estimated value would be based on an estimate of P_{it}.

In implementing this procedure for the PMS data, two complications arose. The first is that for certain years at certain lock chambers SF_{it} is zero. In these cases, the dependent variable in expression 9 is not defined. The second complication is that in certain years the number of lockages at certain lock chambers, L_{it}, is small. In these cases the large sample approximations in expression 10 are not appropriate and so, therefore, neither is the model in expression 9.

The discussion in Kelejian (*3*) suggests that if the number of lockages for each chamber in each time period is large, the first of these problems can be overcome by replacing the dependent variable in expression 9 by

$$Y_{it} = \log_e\{(SF_{it} + (2L_{it})^{-1})/[1 - SF_{it} + (2L_{it})^{-1}]\} \qquad (11)$$

The reason for this is that Y_{it} is defined for all values of SF_{it} in the interval $0 \le SF_{it} \le 1$; furthermore, under reasonable conditions, Y_{it} and $\log_e[SF_{it}/(1 - SF_{it})]$ converge in probability as L_{it} increases beyond limit.

The procedure that was followed in this study is on the basis of a variant of expression 11 and is described in steps detailed in the following. Note that the estimators so obtained are asymptotically efficient because they are equivalent to the corresponding maximum likelihood estimators.

Details of the Procedure

Step 1

Some restriction on the original PMS sample was necessary because the number of lockages in certain years at certain chambers (henceforth, cells) was very small (e.g., as low as

4). The restriction that was imposed was that only data relating to cells for which $L_{it} > Q$ were considered in the estimation procedure. Q was taken as the largest multiple of 50 such that at least ⅔ of the original cells remain in the revised sample. It turned out that $Q = 150$, and the smallest value of L_{it}, say MIN, satisfying $L_{it} > 150$ was MIN = 151. The cut off value of 150 is reasonably large, but not overly restrictive in terms of the scope of the revised sample. For example, the condition $L_{it} > 150$ only eliminates reference to 32 lock chambers, thus leaving 93 such chambers in the sample. The number of cells in the revised sample is 499 which is roughly 67 percent of the original number of cells.

Step 2

Because the number of lockages in each cell of the sample constructed in Step 1 varied from 151 to 1,355, a modified form of Y_{it} in expression 11 was considered; namely

$$^{*}Y_{it} = \log_e\{SF_{it} + (2 \times \text{MIN})^{-1}/[1 - SF_{it} + (2 \times \text{MIN})^{-1}]\} \qquad (12)$$

$^{*}Y_{it}$ was considered for two reasons. First, unlike Y_{it}, $^{*}Y_{it}$ does not induce an artificial variation in the dependent variable, which is due solely to the wide range of values of the number of lockages. Second, there is no penalty in terms of asymptotic efficiency in the use of $^{*}Y_{it}$ as compared with Y_{it} because $^{*}Y_{it}$ and Y_{it} converge as L_{it} increases beyond limit.

Step 3

Taking $^{*}Y_{it}$ as the dependent variable, expression 9 was first estimated by least squares. This provided a consistent estimate of the index I_{it}, say IE_{it}, for each of the 499 cells of the sample.

Step 4

The estimated index, IE_{it}, was then used to obtain an initial but consistent estimate, PE_{it}, of the stall probability P_{it} for each of the 499 cells

$$PE_{it} = \text{EXP}(IE_{it})/[1 + \text{EXP}(IE_{it})] \qquad (13)$$

Correspondingly, the variance of u_{it} as given in expression 10 was then estimated as

$$\hat{\text{v}}\text{ar}(u_{it}) = [L_{it}\, PE_{it}(1 - PE_{it})]^{-1} \qquad (14)$$

Step 5

Finally, with $^{*}Y_{it}$ in expression 12 taken as the dependent variable, expression 9 was reestimated by least squares after deflating each variable by the square root of $\hat{\text{v}}\text{ar}(u_{it})$ in expression 14. This is the feasible generalized least squares procedure.

Let the estimates of the parameters of expression 9 obtained in Step 5 be $b_0, \ldots, b_7, a_1, \ldots, a_5$. Then, because these estimates are based on a consistent and efficient procedure, the final estimate of the stall probability for the ith lock chamber at time t was taken as

$$\hat{P}_{it} = \text{EXP}(\hat{I}_{it})/[1 + \text{EXP}(\hat{I}_{it})] \qquad (15)$$

where

$$\begin{aligned}\hat{I}_{it} = {} & \hat{b}_0 + \hat{b}_1 \text{Age}_{it} + \hat{b}_2 MPT_{it-1} \\ & + \hat{b}_3 ICE_{it-1} + \hat{b}_4 AIT_{it} + \hat{b}_5 \text{Maint}_{it} \\ & + \hat{b}_6 ST_{it-1} + \hat{b}_7 SF_{it-1} + \hat{a}_1 DD1_i \\ & + \hat{a}_2 DD3_i + \hat{a}_3 DD7_i \\ & + \hat{a}_4 DD8_i + \hat{a}_5 DD10_i \end{aligned} \qquad (16)$$

EMPIRICAL RESULTS

Results Relating to the Probability Model

The empirical results obtained by the procedure described in Steps 1 through 5 are given in expression 17. The figures in parentheses beneath the parameter estimates are the absolute values of the corresponding t-ratios. $\hat{r}^2$ is the square of the correlation coefficient between the observed stall frequency, SF_{it}, and its model predicted value $\hat{P}_{it}$ (see expression 15).

$$\begin{aligned}\hat{I}_{it} = {} & -5.956 + .0102\ \text{Age}_{it} + .0071\ MPT_{it-1} \\ & \quad (41.31)\ (5.130) \qquad (2.997) \\ & + .0066\ ICE_{it-1} - .0899\ AIT_{it} \\ & \quad (4.458) \qquad (1.599) \\ & - 1.197\ \text{Maint}_{it} - .0048\ ST_{it-1} + 23.09\ SF_{it-1} \\ & \quad (1.818) \qquad (3.997) \qquad (8.881) \\ & - .2422\ DD1_i + .8219\ DD3_i - .2121\ DD7_i \\ & \quad (2.651) \qquad (7.292) \qquad (2.716) \\ & + .3416\ DD8_i - .3167\ DD10_i\ ;\ \hat{r}^2 = .366 \\ & \quad (3.888) \qquad (3.093) \end{aligned} \qquad (17)$$

The units of measurement underlying expression 17 are: Age is in years; MPT is in minutes per lockage; ICE is in days per year; AIT is in hundreds of minutes; Maint = 1 − EXP(−C) where C is in hundreds of millions of 1982 dollars; SF is the observed stall frequency; ST is in thousands of minutes.

The value of $\hat{r}^2 = .366$ suggests that, overall, the model offers a reasonable explanation of stall probabilities associated with individual lockages. In interpreting this figure one should note that stall probabilities, as measured by stall frequencies, vary widely across lock chambers and time, and therefore are not easily explained. For example, the R^2 statistic (over the sample underlying expression 17) between the annual stall frequency at a lock chamber, and its age is only .015. More extensive results along these lines are given in Kelejian (3). Nevertheless, $\hat{r}^2 = .366$ does imply that 63.4

percent of the variation in stall probabilities is unexplained, and so further studies along these lines could be of value.

On a qualitative level, note that the sign of each estimate given in expression 17 is consistent with prior expectations as described in expression 5. Also note that each of these estimates, if considered alone, is statistically significant at the one-tail .05 level with the sole exception of the coefficient of the average idle time variable. The sign of this coefficient is negative, as anticipated, but its one-tail significance level is .0548. Because strong prior Bayesian beliefs suggest that average idle time is important, and because the one-tail significance level is quite close to .05, the idle time variable was not dropped from the model.

There are no prior sign expectations for the coefficients of the district dummy variables and therefore a test of significance would be determined by a two-tail procedure; clearly the results in expression 17 imply that if these variables are considered individually, each and every one of them would be statistically significant at the two-tail .05 level. The joint significance of the district dummy variables is confirmed by the corresponding F test, namely $\hat{F} = 15.18 > F(.95/5,486) = 2.23$.

Districts that are represented in the sample but for which there are no dummy variables in expression 17 are Mobile, Walla Walla, Louisville, St. Louis, Little Rock, and Seattle. Therefore, if a coefficient corresponding to a dummy variable in expression 17 is positive, the stall probability in the corresponding district is higher than in the excluded 5 districts for given and equal values of the other variables in expression 17. Districts 3 and 8 (Nashville and Huntington) fall into this category. Similarly, if such a coefficient in expression 17 is negative, the stall probability in the corresponding district is lower than in the excluded five districts for given and equal values of the other variables in expression 17. Districts 1, 7, and 10 (Pittsburgh, Rock Island, and St. Paul) fall into this category.

One measure of the magnitude of these district effects is the consequent change in the stall probability. For example, in District 1, (Pittsburgh), the sample mean of the index in expression 17 is $I_1 = -5.447$; the corresponding stall probability is $\hat{P}_1 = .00429$. If District 1 were typical, as say described by the five excluded districts, the coefficient of its dummy variable would be zero. In this case, the sample mean of its index would be $IE_1 = -5.2048$, and the corresponding stall probability would be $PE_1 = .00546$. Therefore, whatever the special effects associated with District 1, they lead to a reduction of .00117 in the stall probability. Since these probabilities are small, this small change represents a large percentage change. Specifically, taking $(\hat{P}_1 + PE_1)/2$ as the base, the district effect (at the sample mean) associated with District 1 leads to a 24 percent reduction in the stall probability. Corresponding figures for Districts 3, 7, 8, and 10 are given in Table 3. Consistent with the results for District 1, a glance at the table suggests that these districts also have effects that are important in percentage terms concerning stall probabilities.

Further results relating to the empirical model are given in Table 4. Specifically, the table gives the stall probability corresponding to sample mean values of the variables determining the index in expression 17. This figure, namely .0056, can be interpreted as the probability that the average or typical chamber will have a stall on a given lockage. Conversely, the probability that a stall will not take place at such a typical chamber is .9944. This probability is so high that even if a reasonably large number of lockages take place over a given period of time, the probability that a stall will not occur during that time could remain non-negligible.

TABLE 3 DISTRICT EFFECTS AT SAMPLE MEAN VALUES

District	Probability Changes	Probability Change (%)
Nashville	.0084	77
Huntington	.0041	50
Rock Island	-.0013	-21
Pittsburgh	-.0012	-24
St. Paul	-.0043	-31

TABLE 4 STALL PROBABILITIES AND ELASTICITIES

Stall Probability			Elasticities					
Lowest	Highest	Mean	AGE	MPT	ICE	AIT	MAINT	ST
.0023	.0374	.0056	.386	.287	.106	-.079	-.009	-.027

The table also gives the lowest and highest values of the stall probability based on the values of the index over the chambers and years in the sample. These figures, namely .0023 and .0374, correspond, respectively, to Chamber 1 of the Old River Lock on the Mississippi River (ORLMR) for 1982, and Chamber 1 at the Gallipolis Locks and Dam on the Ohio River (GLDOR) in 1986. These figures differ by more than a factor of ten. As an indication of time variation, the stall probability at the ORLMR for 1986 is .0025; the stall probability is .0096 for 1982 at the GLDOR. Among other things, these results suggest that stall potentials, as measured by stall probabilities, vary considerably from chamber to chamber, as well as over time. Given the results in expression 17, and the model in expression 15, the stall probability can be calculated for any chamber, for any year, as long as the values of the independent variables are known. Clearly, the calculation of such stall probabilities should be helpful in allocating scarce major maintenance funds.

Table 4 also gives estimates of the elasticities of the stall probability with respect to six of the index variables, again at sample mean values. These elasticities were calculated as

$$d\log_e(P_{it})/d\log_e(Z^j_{it}) = \bar{Z}^j \hat{b}_j/(1 + \mathrm{EXP}(\bar{I})),\ j = 1, \ldots, 6 \qquad (18)$$

where Z^j_{it} is the jth explanatory variable (excluding the intercept) in expression 16; $\hat{b}_j$ is its corresponding estimated coefficient given in expression 17, and $\bar{Z}^j$ and $\bar{I}$ are the sample averages of Z^j_{it} and $\hat{I}_{it}$.

The elasticities in Table 4 indicate the relative sensitivity of the stall probability with respect to a given percentage change in the value of the corresponding explanatory variable at sample mean values. For example, the elasticity with respect to the age variable is .386. This figure suggests that, at sample mean values, a 1 percent (a 10 percent) increase in the age of a chamber would (other things equal) lead to a .386 percent (a 3.86 percent) increase in the stall probability Among other things, the figures in Table 4 suggest that stall probabilities for a typical lock chamber are more sensitive to small percentage changes in the age of the chamber, than to small percentage changes in the other variables of the index.

Figures 1 through 4 give further insights concerning the probability model. Figure 1 describes the relationship between the stall probability and the age of the chamber at sample mean values of the other variables involved in the index. Again, since these sample mean values could be viewed as typical, Figure 1 essentially describes a time profile of a stall probability for a typical chamber. As the chamber ages, the probability increases. Calculations based on the diagram suggest that this probability is roughly 20 percent higher when the chamber is 60 as compared with 40 years old.

Figure 2 describes the relationship between the stall probability and the extent of major maintenance, as measured by its 1982 dollar cost, again at sample mean values. As expected, the more extensive the maintenance, the lower the probability. Calculations performed on the basis of the diagram suggest that, for a typical chamber, a 30 million 1982 dollar major maintenance reduces the stall probability by, roughly, 35 percent. Similarly, a 20 million 1982 dollar major maintenance reduces this probability by, roughly, 20 percent. Figure 3 describes the relationship between the stall probability and mean processing time, at sample mean values. The figure suggests that, for the typical chamber, stall probabilities are roughly 45 percent more likely when the mean processing time is 100 minutes per lockage than when it is 40 minutes per lockage.

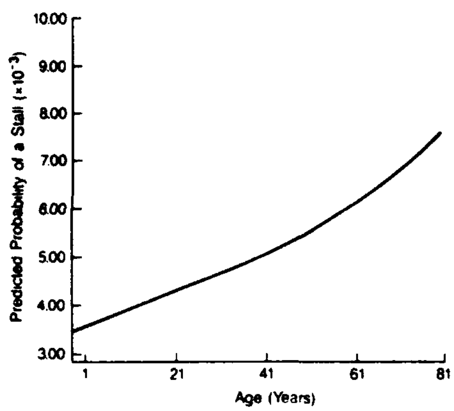

FIGURE 1 Effect of age on the predicted probability of a stall.

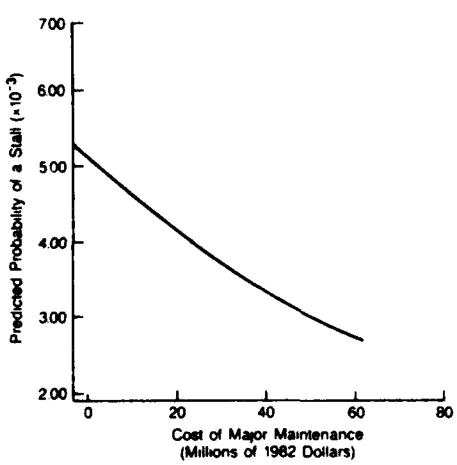

FIGURE 2 Effect of major maintenance on the predicted probability of a stall.

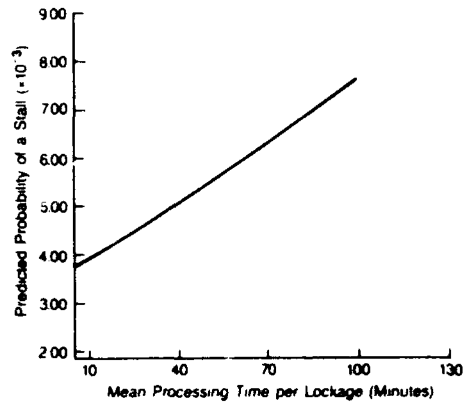

FIGURE 3 Effect of mean processing time on the predicted probability of a stall.

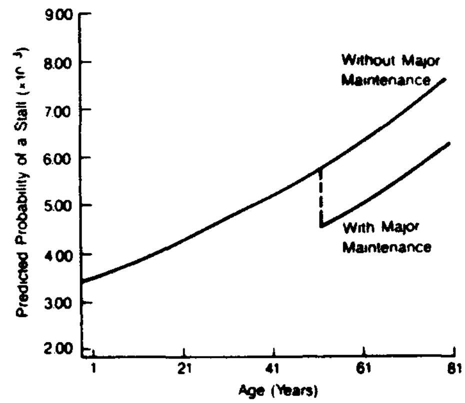

FIGURE 4 Effect of major maintenance on the effective age of a lock.

Finally, Figure 4 describes how major maintenance reduces the effective age of a chamber. The upper curve in that figure outlines the relationship between the stall probability and the age of the chamber if there is no major maintenance and the other relevant variables are equal to their sample means. The lower curve describes the change in the probabilities outlined by the upper curve if a 20 million 1982 dollar major maintenance were undertaken when the chamber is 50 years old. A 20 million 1982 dollar major maintenance was considered in this illustration because it is, roughly, the average cost of such maintenance completed during or before 1987.

If the lower curve, at any age exceeding 50 years, is horizontally extended to the left, it will intersect the upper curve corresponding to an age which is, roughly, between 20 and 25 years earlier. The suggestion is that, for the typical chamber, a 20 million 1982 dollar major maintenance, undertaken when the chamber is 50 years old, reduces the effective age of that chamber by, roughly, 20 to 25 years.

The reduction in the effective age of a particular chamber corresponding to a proposed major maintenance of a certain dollar magnitude can be done in a similar, but more exact way. Specifically, let $\hat{I}^B_{it}$ be the value of the index in expression 17 for lock Chamber *i* at time *t* before the major maintenance. Let $\hat{I}^A_{it}$ be the value of that index after the major maintenance. Because the index is reduced if major maintenance is undertaken, $\hat{I}^A_{it} < \hat{I}^B_{it}$ and so the "after" stall probability would be less than the "before" stall probability. The effective age of the chamber after the maintenance is the value of the age variable that equates the before index, $\hat{I}^B_{it}$, to the after index, $\hat{I}^A_{it}$. That is, let I^B_{it} be the net sum of the right hand side of expression 17, before the major maintenance, with the exception of the age variable: $\hat{I}^B_{it} = I^B_{it} + .0102 * Age_{it}$. Then the effective age of chamber *i* at time *t* is Age^E_{it} where

$$Age^E_{it} = (\hat{I}^A_{it} - I^B_{it})/.0102 \tag{19}$$

The reduction in the effective age is therefore $Age_{it} - Age^E_{it}$.

Suggestions Concerning Further Calculations

Calculations concerning chambers in districts which are not in the sample require an assumption concerning the district effect as described in the index (see expression 17). One possibility is that Corps personnel could use expert opinion to determine the district effect; given this, stall probabilities could be evaluated for any chamber of interest, for any year, as long as the values of the variables determining the index in expression 17 are known. The magnitude of the district effects of Districts 1, 3, 7, 8, and 10 should offer guidance if this route is taken.

Another possibility is to assume that the district effect corresponding to a chamber of interest, which is not in the sample, is equal to the average of the effects of those for Districts 1, 3, 7, 8, and 10. Still another possibility is to consider worst and best case scenarios. For example, the district effect could be taken to be equal to that of District 3, which would be a worst case scenario. Given this, a policy could be evaluated in terms of its effect on the stall probability. The district effect could then be taken to be equal to that of District 10, which would be a best case scenario. Given this, the policy could again be evaluated. Comparisons between the two cases should be of interest.

SUMMARY AND CONCLUSIONS

A probability model of lock failures has been presented. The qualitative results corresponding to the underlying variables are consistent with expectations. The usefulness and flexibility of the model in evaluating changes in the values of these variables has been demonstrated. Among other things, this demonstration suggests that many major maintenance projects relating to lock chambers can be evaluated in terms of their consequent effect on lock failure probabilities. It was also demonstrated that the extent of the renewal of a chamber in response to major maintenance can be calculated.

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The views expressed in this study are solely those of the author, and are not necessarily those of the Army Corps.

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EFFECTS OF LOCK INTERDEPENDENCE ON TOW DELAYS

by

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Effects of Lock Interdependence on Tow Delays

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Introduction

This study examines the effect on tow delays of interactions among successive locks. Relatively simple queueing models may be used for analyzing single-lock delays. However, considerable interdependence may exist among locks in a series, especially if the locks are relatively closely spaced and if congestion levels are high. The tow departure distributions at a lock differ from the arrival distributions at that lock since the service-time distributions change the tow headways. The departures from one lock usually affect the arrivals at the next lock. Thus, it is risky to assume that the locks are independent. The interdependence among locks increases the difficulty in estimating delays for a system of locks, since it is necessary at each lock to identify the interarrival-time distributions of flows from adjacent locks.

Analysis Method

A simulation model which reflects the interaction among locks is used here to compare results obtained by assuming (a) isolated locks or (b) interdependent locks. This model (Dai 1991, Dai and Schonfeld 1991) is microscopic, event scanning, and programmed in FORTRAN. Its most detailed documentation is provided in Dai (1991). By running the model with actual distances among locks, interdependence effects are automatically considered. As distances among locks are artificially increased, the locks approach an isolated condition. Isolation may also be modeled by generating tow arrivals at each lock according to a Poisson distribution, which is equivalent to generating exponential interarrival times. As the distances between successive locks increase, and as the effect of the departure distributions is gradually randomized, the arrivals should asymptotically approach a Poisson distribution.

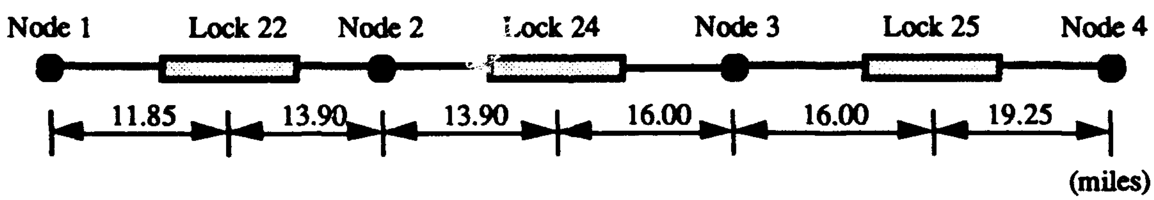


Figure 1
The Geometric Configuration for the Segment on the Mississippi River

1987 PMS data from Mississippi Locks 22, 24, and 25 were used in the experiments reported here. The geometric configuration of this system is shown in Figure 1. Four nodes and three locks are included in this system. The average trip rate differed for each lock and is shown in Table 1. Results have been compared for two different traffic volumes representing (1) 1987 levels (from PMS data), which correspond to a 0.65 V/C (volume/capacity) ratio at the critical Lock 22, and (2) a more congested hypothetical 0.95 V/C ratio at Lock 22. For the 0.95 V/C ratio case, all volumes are increased by the same multiplier (0.95/0.65) as shown in Figures 2 and 3. Traffic volumes are equal in the two directions.

Lock No.	Lock 22	Lock 24	Lock 25
Trip Rate (65%)	5.25	5.5	5.87
Trip Rate (95%)	7.67	8.03	8.57

(trips/day)

Table 1. Trip Rates for Each Lock

The trip rates among the various O/D pairs which actually exchange traffic are shown in Table 2.

O/D pair No.	1-4	1-2	2-3	3-4
Trip Rate (65%)	5.08	0.17	0.42	0.79
Trip Rate (95%)	7.42	0.25	0.61	1.15

(trips/day)

Table 2. Trip Rates for Each O/D Pair

** 1-4 means from node 1 to node 4

The actual node distances are 25.75, 29.90, and 35.25 miles form node 1 to node 2, from node 2 to node 3, and from node 3 to node 4, respectively. The average numbers of stall events are 30, 40, and 49 per year at Locks 22, 24, and 25, respectively. The average tow speed is 203.76 miles per day with a standard deviation 81.36 miles per day. Time periods of 2100 days are simulated, after a steady state condition is reached. At each lock, the data collected include the directional interdeparture-time distributions, the overall interarrival-time distributions, and the average wait time. The corresponding trip rate, V/C ratio, lock service time distribution, distance and the tow speed distribution were also recorded. To obtain the required information, the simulation program was run for 30 independent replications in each relevant case, i.e. 30x2100 days. To insure that the results were for a steady state, each simulation run discarded the first 20,000 observations and collected the next 40,000 values for evaluating the results.

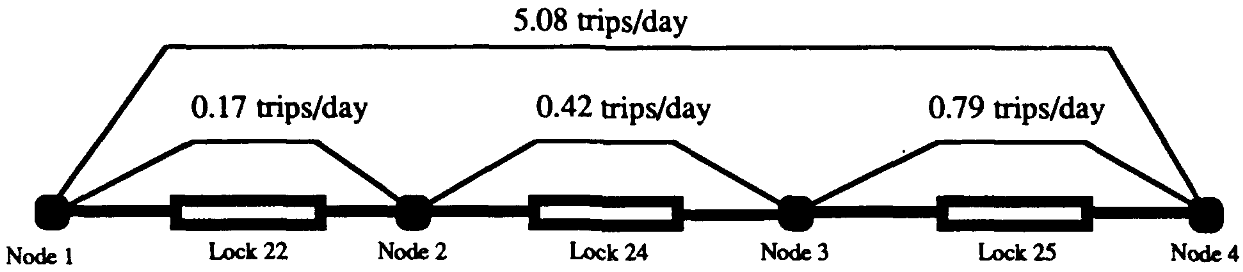


Figure 2
The Average Trip Rate at 65% Service Capacity of the Bottleneck Lock Both Directions

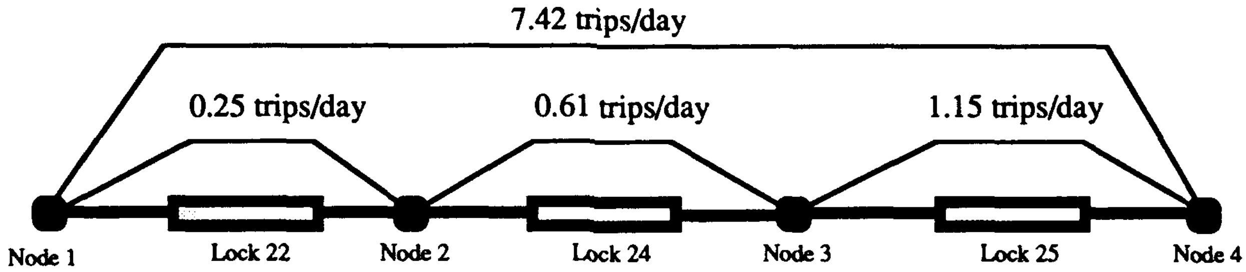


Figure 3
The Average Trip Rate at 95% Service Capacity of the Bottleneck Lock Both Directions

Results

The overall results obtained from running the simulation model are shown in Appendices 1 and 2. The average delays, standard deviations, and standard errors of the mean for different trip rates and lock distance are summarized in Tables 3 and 4.

Distance Multiplier	Lock 22	Lock 24	Lock 25	Total System
0.1	101.92* (4.26)** [0.78]***	76.95 (2.87) [0.52]	86.70 (3.72) [0.68]	265.67 (7.97) [1.46]
1	106.80 (4.53) [0.83]	83.29 (2.84) [0.52]	90.70 (3.75) [0.68]	280.79 (8.00) [1.46]
10	113.87 (3.50) [0.64]	95.58 (3.58) [0.65]	96.74 (3.70) [0.68]	306.19 (7.64) [1.40]
100	110.69 (4.19) [0.76]	96.71 (3.10) [0.57]	95.82 (2.77) [0.51]	303.22 (7.28) [1.33]

* : mean value
** : standard deviation
***: standard error of the mean

Table 3 Effects of Lock Separation on Delays (V/C = 0.65 at Lock 22)

Distance Multiplier	Lock 22	Lock 24	Lock 25	Total System
0.1	716.94 (74.87) [13.67]	454.40 (50.20) [9.16]	406.84 (41.82) [7.63]	1578.18 (123.98) [22.64]
1	737.39 (72.63) [13.26]	475.86 (52.65) [9.61]	430.07 (42.55) [7.77]	1643.32 (122.10) [22.29]
10	805.62 (85.66) [15.64]	543.84 (55.99) [10.22]	489.51 (44.64) [8.15]	1838.97 (134.66) [24.59]
100	851.98 (82.19) [15.01]	647.16 (64.52) [11.78]	534.71 (49.78) [9.09]	2043.85 (140.93) [25.73]
200	847.04 (100.56) [18.36]	659.31 (74.89) [13.67]	526.84 (52.26) [9.54]	2043.19 (158.11) [28.87]

Table 4 Effects of Lock Separation on Delays (V/C = 0.95 at Lock 22)

The distance multipliers are used to vary all inter-lock distances proportionally. 1 represents actual distances. Distance multipliers of 100 or 200 are, for all practical purposes, equivalent to infinite distances, i.e. complete isolation of individual locks.

The results in Tables 1 and 2 confirm, as expected, that delays asymptotically approach the value for isolated locks as distances increase and interdependence becomes negligible. However, at actual distances, considerable errors in delay estimation would be made if the interdependence is neglected. To test whether the locks are essentially isolated (i.e. whether the arrivals are Poisson distributed), the tow interarrival times at each lock have been extracted and analyzed.

When the distance multiplier reaches 100 at a 65% volume/capacity ratio at Lock 22, the average tow delays converge to 111, 97, and 96 minutes per tow at Locks 22, 24, and 25, respectively. Similarly, when the distance multiplier reaches 200 at the 95% V/C ratio at Lock 22, the average tow delays converge to 850, 650, and 530 minutes per tow at Locks 22, 24, 25, respectively. Furthermore, the average system delay per tow converges to 303 and 2040 minutes per tow at 65% and 95% V/C ratios, respectively. We found that at large distance multipliers (i.e. 100 or 200) the tow interarrival times at each lock are indeed exponentially distributed, and the numbers of tow arrival events in every small interval is Poisson distributed, as would be theoretically expected if the locks were isolated and independent.

The results in Table 5 show that for individual locks in this system, the average delays are overestimated from 4% (Lock V/C=0.65) to 39% (Lock 24, V/C=0.95). For the entire 3-lock series the overestimation error is 8% at V/C = 0.65 and 24% at V/C = 0.95. As expected, the results confirm that delay overestimation errors increase significantly as the system approaches capacity. Therefore it is very important to explicitly consider the effects of interdependence among locks, especially as congestion increases and/or when locks are relatively closely spaced. Based on the results in Tables 3 and 4, the average tow delays at each lock have been plotted in Figures 4, 5, and 6. The effect of lock separations on combined delays in the 3-lock system is shown Figure 7. It clearly shows that the interdependence effects, and the errors resulting from neglecting them are much greater as congestion increases in the system. The standard deviation o f combined delays for various V/C ratios (5%, 35%, 65%, and 95%) at Lock 22, 24, and 25 is shown in Figure 8. It points out that when the congestion become serious at the bottleneck lock, the standard deviation of the tow delays increase drastically.

Lock	V/C = 0.65 at Lock 22		V/C = 0.95 at Lock 22	
	Interdependence*	Isolation**	Interdependence	Isolation ***
Lock 22	106.80 (100%)	110.69 (104%)	737.39 (100%)	847.04 (115%)
Lock 24	83.29 (100%)	96.71 (116%)	475.86 (100%)	659.31 (139%)
Lock 25	90.70 (100%)	95.82 (106%)	430.07 (100%)	526.84 (122%)
Total System	280.79 (100%)	303.22 (108%)	1643.32 (100%)	2043.19 (124%)

* Interdependence : actual distances among locks
** Isolation at V/C = 0.65 : 100 x (actual distances among locks)
*** Isolation at V/C = 0.95 : 200 x (actual distances among locks)

Table 5 Summry Result for Delay Estimation

Conclusion

The results of this analysis are simply summarized in Table 5 and in Figure 7. They show that lock interactions may be quite significant and that considerable errors may be introduced when such interdependence is neglected for the sake of simplified models. The interdependence effects, and the associated errors of neglecting them increase (1) as locks are located closer together and (2) as congestion levels increase in the system.

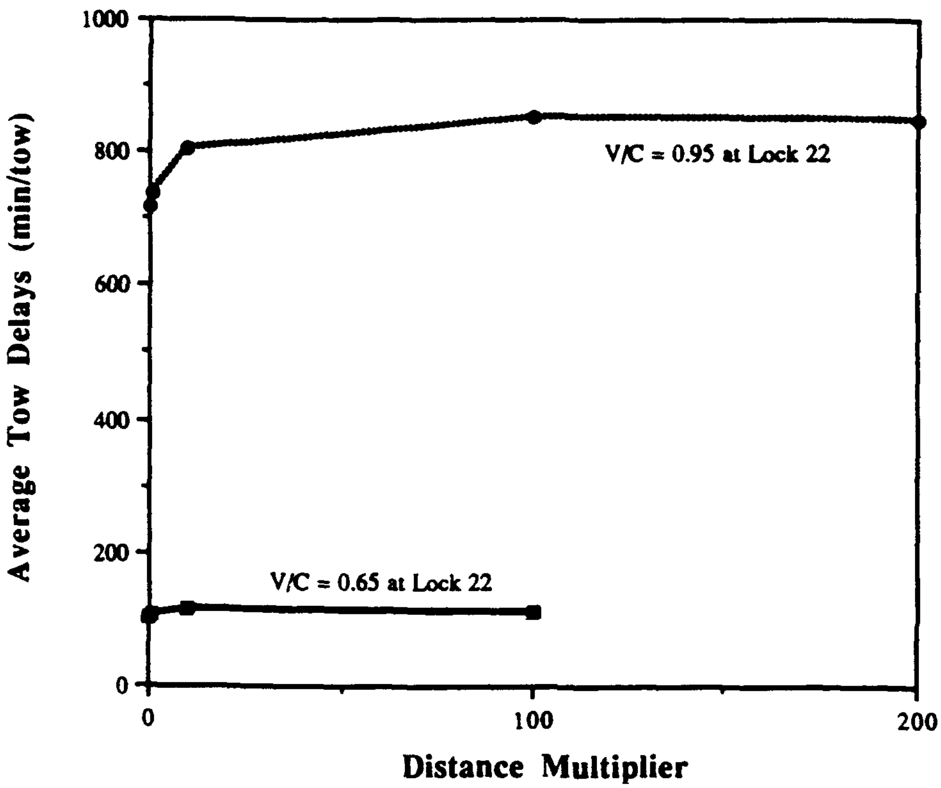


Figure 4
Average Tow Delays for Different Trip Rates at Lock 22 on Mississippi River

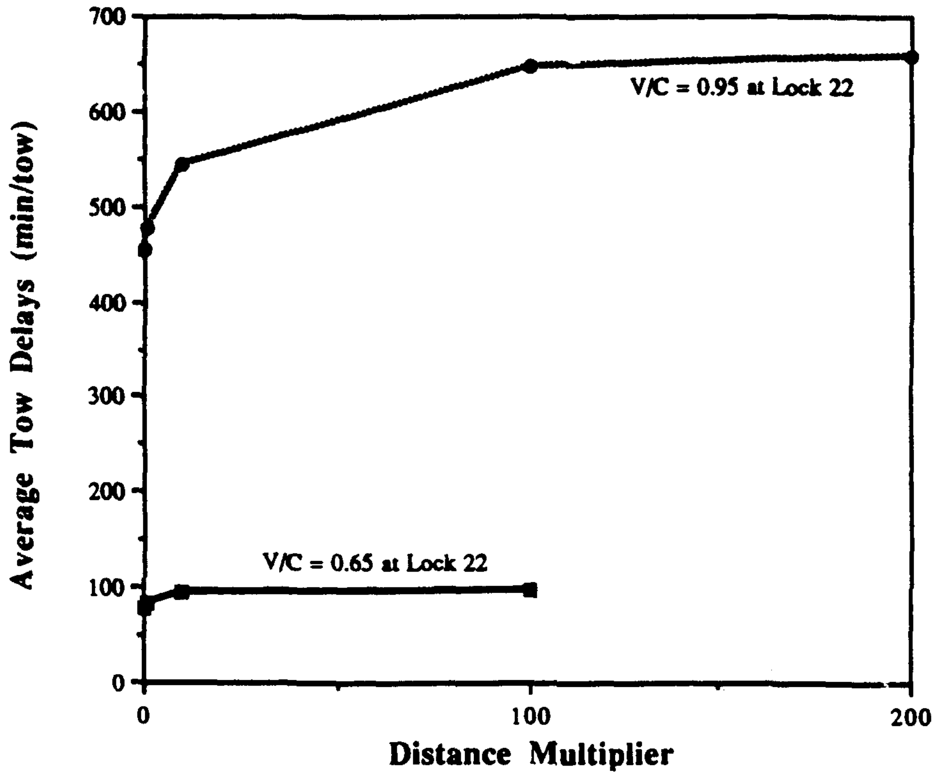


Figure 5
Average Tow Delays for Different Trip Rates at Lock 24 on Mississippi River

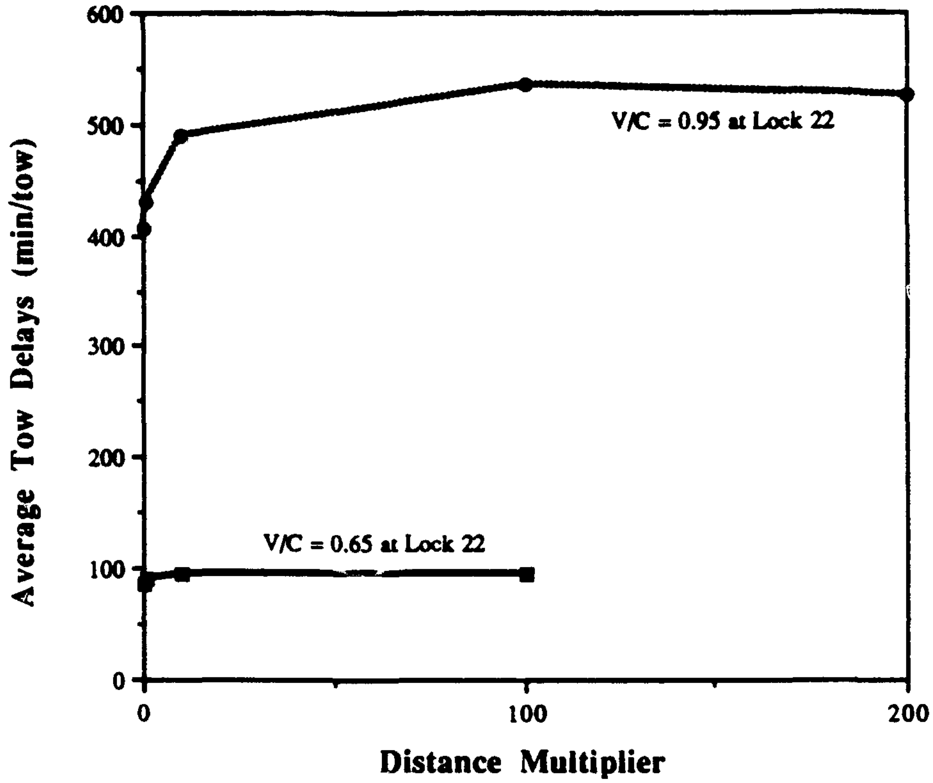


Figure 6
Average Tow Delays for Different Trip Rates at Lock 25 on Mississippi River

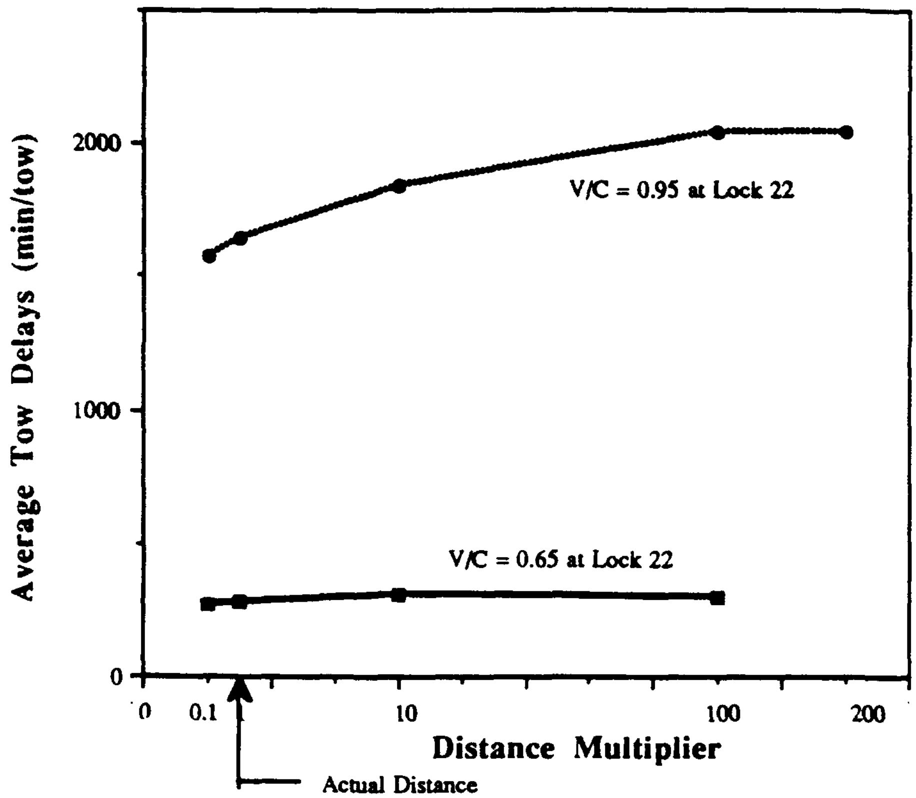
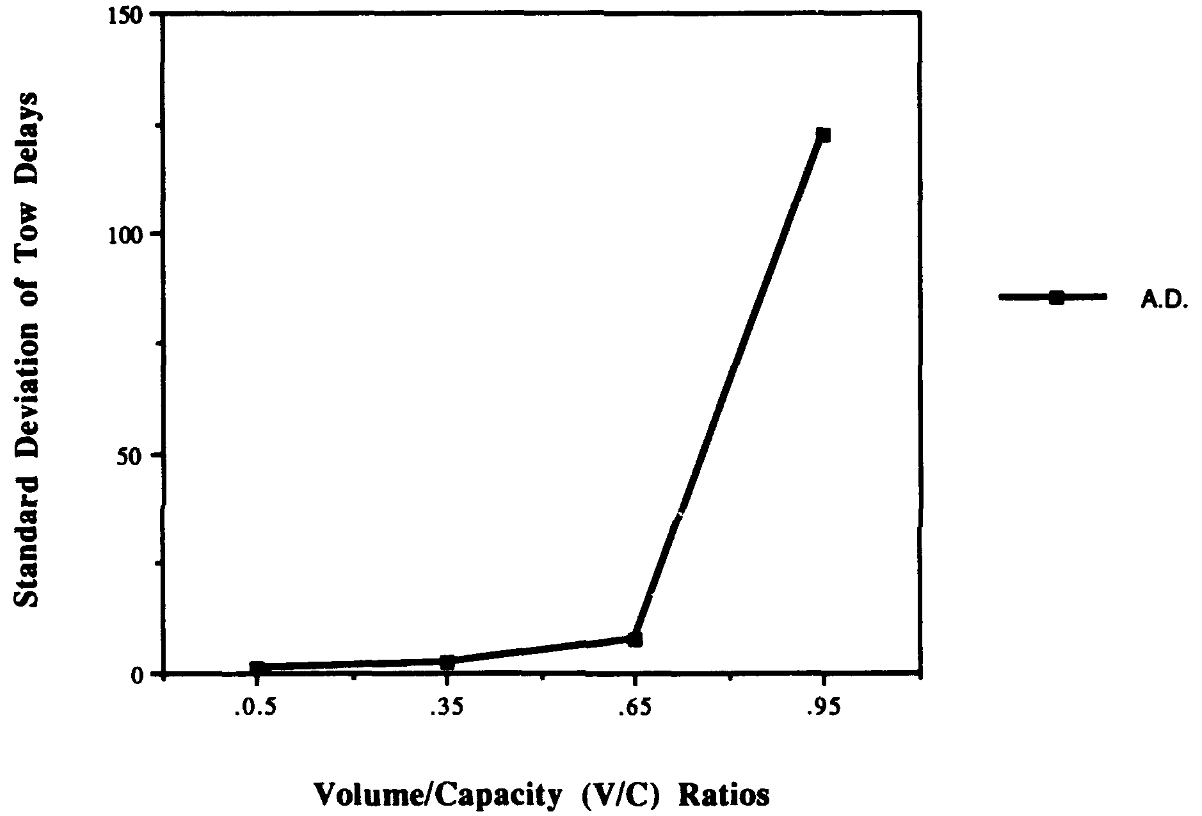


Figure 7
Effect of Lock Separations on Combined Delays at Locks 22, 24, and 25

Figure 8
Standard Deviations of Combined Delays
for Various V/C Ratios at Lock 22, 24, and 25



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Appendix 1

Simulated average tow delays at 5% of capacity (30 replications)

(1) Trip Rate : 5% of capacity at Lock 22

Distance : 0.1 x actual distance

lock 22	lock 24	lock 25	Total System
6.22	3.97	6.97	17.16
6.28	4.61	7.63	18.52
5.85	5.43	6.52	17.80
5.31	3.78	7.40	16.49
6.03	4.98	7.54	18.55
5.45	3.03	6.57	15.05
6.25	3.77	7.26	17.28
4.99	4.22	5.67	14.88
4.34	3.63	7.21	15.18
4.82	3.93	8.62	17.37
5.09	4.75	6.86	16.70
9.66	3.67	7.07	20.40
4.89	4.36	7.01	16.26
3.85	3.88	8.67	16.40
6.58	3.21	7.76	17.55
4.66	3.56	6.64	14.86
4.65	4.12	7.89	16.66
6.67	3.46	11.92	22.05
5.29	4.23	6.18	15.70
5.51	5.15	8.02	18.68
4.84	3.37	5.31	13.52
5.83	3.09	5.83	14.75
6.67	3.44	5.96	16.07
5.82	4.18	6.50	16.50
6.20	6.38	6.08	18.66
4.74	4.46	7.58	16.78
5.26	3.80	7.76	16.82
4.56	3.05	7.25	14.86
6.04	3.77	8.31	18.12
5.50	3.29	6.91	15.70
mean	mean	mean	mean
5.59	4.02	7.23	16.84
std	std	std	std
1.05	.75	1.20	1.76
std error	std error	std error	std erro
.19	.14	.22	.32

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(2) Trip Rate : 5% of capacity at Lock 22
Distance : 1 x actual distance

lock 22	lock 24	lock 25	Total System
5.14	3.40	6.43	14.97
5.90	4.09	6.52	16.51
6.26	3.58	6.23	16.07
5.07	3.36	7.67	16.10
6.41	4.35	6.31	17.07
4.23	3.91	4.95	13.09
5.34	3.48	8.25	17.07
6.51	4.51	5.59	16.61
5.25	4.07	9.06	18.38
4.73	3.50	7.09	15.32
5.43	4.24	6.22	15.89
7.87	3.48	6.82	18.17
5.89	4.03	6.95	16.87
3.57	3.43	6.46	13.46
5.71	3.70	7.84	17.25
5.28	3.94	6.00	15.22
4.85	3.42	7.89	16.16
5.40	4.68	9.18	19.26
5.38	4.23	6.15	15.76
4.53	3.45	7.27	15.25
4.52	3.18	6.39	14.09
5.45	3.10	5.30	13.85
6.87	4.02	6.56	17.45
5.54	4.55	7.92	18.01
6.20	4.81	7.52	18.53
5.50	4.99	5.47	15.96
5.97	3.39	5.40	14.76
4.00	4.15	8.24	16.39
5.86	3.52	8.17	17.55
5.09	4.60	6.68	16.37
mean	mean	mean	mean
5.46	3.91	6.88	16.25
std	std	std	std
.86	.51	1.08	1.50
std error	std error	std error	std error
.16	.09	.20	.27

* mean : average value
* std : standard deviation
* std error : standard error of the mean

(3) Trip Rate : 5% of capacity at Lock 22

Distance : 10 x actual distance

lock 22	lock 24	lock 25	Total System
5.56	3.49	7.99	17.04
7.19	3.99	8.77	19.95
5.50	3.51	7.97	16.98
4.16	3.30	6.65	14.11
6.69	5.15	7.35	19.19
6.03	3.92	5.88	15.83
4.07	3.92	9.32	17.31
6.62	4.24	6.77	17.63
6.09	3.59	7.73	17.41
7.04	4.26	5.76	17.06
5.48	3.30	6.83	15.61
5.70	4.37	8.79	18.86
5.28	4.63	6.44	16.35
5.19	4.06	7.35	16.60
5.71	3.94	6.81	16.46
8.09	4.52	5.98	18.59
4.69	3.90	7.82	16.41
6.76	3.00	6.41	16.17
5.38	5.01	6.34	16.73
6.71	3.53	7.59	17.83
5.93	4.61	7.05	17.59
5.40	5.83	7.24	18.47
6.14	4.05	8.13	18.32
4.93	4.95	5.69	15.57
5.87	4.55	6.75	17.17
5.36	3.68	4.08	13.12
6.54	4.23	6.18	16.95
4.95	3.85	7.30	16.10
3.42	3.33	11.68	18.43
5.11	5.08	8.59	18.78
mean	mean	mean	mean
5.72	4.13	7.24	17.09
std	std	std	std
.98	.64	1.36	1.43
std error	std error	std error	std error
.18	.12	.25	.26

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(4) **Trip Rate : 5% of capacity at Lock 22**
Distance : 100 x actual distance

lock 22	lock 24	lock 25	Total System
4.13	3.84	7.77	15.74
6.75	4.64	5.82	17.21
6.10	4.49	9.87	20.46
5.72	3.77	8.20	17.69
4.84	4.51	5.93	15.28
5.65	3.30	7.71	16.66
4.42	4.16	8.00	16.58
3.93	3.80	5.45	13.18
5.75	5.00	5.58	16.33
6.76	3.28	6.02	16.06
6.39	3.39	7.09	16.87
8.39	3.46	4.99	16.84
4.66	4.73	4.88	14.27
4.84	3.37	7.49	15.70
3.61	3.15	5.68	12.44
4.29	5.29	5.08	14.66
6.05	3.46	7.49	17.00
6.44	4.52	9.55	20.51
5.13	4.06	5.79	14.98
5.64	3.79	8.22	17.65
6.79	3.34	5.38	15.51
6.76	4.63	5.95	17.34
5.91	4.63	7.43	17.97
5.96	3.55	6.80	16.31
5.55	3.69	6.20	15.44
5.38	3.40	5.57	14.35
5.71	4.94	7.09	17.74
4.28	2.90	9.57	16.75
6.35	3.74	7.80	17.89
3.90	4.76	9.72	18.38
mean	mean	mean	mean
5.54	3.99	6.94	16.46
std	std	std	std
1.08	.64	1.47	1.76
std error	std error	std error	std error
.20	.12	.27	.32

* mean : average value

* std : standard deviation

* std error : standard error of the mean

Appendix 2

Simulated average tow delays at 5% of capacity (30 replications)

(1) Trip Rate : 35% of capacity at Lock 22

Distance : 0.1 x actual distance

lock 22	lock 24	lock 25	Total System
32.30	26.42	32.35	91.07
36.25	26.26	30.49	93.00
33.59	27.11	32.83	93.53
36.73	26.99	30.98	94.70
34.10	28.01	30.50	92.61
33.43	27.29	31.52	92.24
35.54	27.07	34.30	96.91
31.96	26.10	29.10	87.16
37.85	27.05	31.38	96.28
33.87	27.90	30.07	91.84
34.85	26.63	31.17	92.65
35.22	25.79	32.59	93.60
34.83	28.31	32.18	95.32
33.61	26.28	31.72	91.61
34.49	27.25	35.09	96.83
33.43	26.70	31.02	91.15
35.53	25.73	31.88	93.14
37.89	27.84	32.30	98.03
32.53	25.54	29.85	87.92
33.28	27.12	35.31	95.71
31.63	24.85	30.27	86.75
36.41	26.52	31.91	94.84
34.65	27.05	31.23	92.93
34.08	27.58	30.68	92.34
35.57	28.57	31.77	95.91
34.68	27.20	33.53	95.41
37.96	27.23	30.36	95.55
35.81	27.64	34.54	97.99
34.37	27.35	32.83	94.55
34.89	27.13	32.25	94.27
mean 34.71	mean 26.95	mean 31.87	mean 93.53
std 1.64	std .82	std 1.52	std 2.81
std error .30	std error .15	std error .28	std error .51

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(2) Trip Rate : 35% of capacity at Lock 22
Distance : 1 x actual distance

lock 22	lock 24	lock 25	Total System
34.66	28.37	33.38	96.41
35.91	29.09	30.67	95.67
32.89	28.11	33.88	94.88
36.07	28.43	31.80	96.30
35.93	29.45	30.74	96.12
33.53	27.22	31.78	92.53
34.37	28.83	34.65	97.85
32.59	27.63	31.04	91.26
36.19	27.66	31.88	95.73
36.36	27.80	30.68	94.84
35.13	28.40	30.32	93.85
34.71	28.40	34.05	97.16
35.19	30.20	32.98	98.37
33.57	27.49	33.54	94.60
36.36	27.88	37.03	101.27
34.05	29.56	31.25	94.86
35.05	27.72	33.40	96.17
37.63	29.95	34.02	101.60
33.97	27.46	30.18	91.61
35.17	29.26	35.14	99.57
33.46	27.06	31.49	92.01
36.85	28.32	31.67	96.84
35.38	27.51	31.28	94.17
33.08	28.81	30.94	92.83
35.30	29.73	32.95	97.98
35.39	29.07	33.11	97.57
36.99	29.59	31.46	98.04
34.50	28.74	33.78	97.02
33.44	28.14	34.95	96.53
36.03	28.22	33.07	97.32
mean	mean	mean	mean
34.99	28.47	32.57	96.03
std	std	std	std
1.29	.84	1.65	2.52
std error	std error	std error	std error
.23	.15	.30	.46

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(3) **Trip Rate : 35% of capacity at Lock 22**

Distance : 10 x actual distance

lock 22	lock 24	lock 25	Total System
33.96	30.18	33.93	98.07
35.32	31.41	30.43	97.16
35.01	30.31	35.61	100.93
36.86	29.36	32.02	98.24
37.32	31.06	30.00	98.38
34.29	29.68	31.32	95.29
34.38	29.98	32.78	97.14
35.53	30.89	32.50	98.92
38.12	29.52	33.09	100.73
35.30	28.27	31.74	95.31
34.35	30.75	31.30	96.40
35.17	27.65	34.71	97.53
33.22	32.11	33.63	98.96
33.95	30.38	34.05	98.38
35.88	29.83	35.49	101.20
34.49	29.04	30.63	94.16
36.28	33.03	31.12	100.43
36.79	30.28	33.27	100.34
34.95	30.39	30.42	95.76
34.81	30.35	35.43	100.59
33.29	28.85	32.14	94.28
37.68	28.90	34.56	101.14
36.29	28.71	33.90	98.90
35.12	28.34	33.03	96.49
35.16	29.77	32.49	97.42
35.35	30.33	30.68	96.36
39.34	30.44	32.41	102.19
36.78	29.40	33.66	99.84
35.81	29.71	35.41	100.93
35.91	30.01	33.38	99.30
mean 35.56	mean 29.96	mean 32.84	mean 98.36
std 1.39	std 1.11	std 1.63	std 2.18
std error .25	std error .20	std error .30	std error .40

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(4) Trip Rate : 35% of capacity at Lock 22

Distance : 100 x actual distance

lock 22	lock 24	lock 25	Total System
36.20	29.69	33.61	99.50
37.88	30.66	31.68	100.22
34.45	28.78	33.33	96.56
34.40	30.58	33.27	98.25
37.92	33.25	33.01	104.18
33.27	28.67	32.15	94.09
35.50	28.80	35.40	99.70
35.11	28.87	31.01	94.99
33.86	29.62	33.94	97.42
34.86	28.76	33.32	96.94
34.58	30.52	31.66	96.76
35.80	30.39	34.08	100.27
35.60	30.81	34.44	100.85
32.92	27.09	33.69	93.70
36.56	30.69	34.55	101.80
32.75	28.52	32.18	93.45
38.03	30.92	31.05	100.00
36.93	31.84	33.25	102.02
35.96	30.67	30.70	97.33
34.88	31.08	34.42	100.38
32.86	27.80	31.90	92.56
36.11	29.63	33.43	99.17
36.24	30.15	32.13	98.52
35.28	29.98	32.07	97.33
35.67	28.67	32.17	96.51
34.21	30.06	31.23	95.50
38.75	28.86	31.01	98.62
35.45	30.80	32.59	98.84
34.12	29.99	35.60	99.71
33.89	30.87	30.45	95.21
mean	mean	mean	mean
35.33	29.90	32.78	98.01
std	std	std	std
1.55	1.24	1.37	2.73
std error	std error	std error	std error
.28	.23	.25	.50

* mean : average value
* std : standard deviation
* std error : standard error of the mean

Appendix 3

Simulated average tow delays at 65% of capacity (30 replications)

(1) Trip Rate : 65% of capacity at Lock 22

Distance : 0.1 x actual distance

lock 22	lock 24	lock 25	Total System
101.48	79.78	90.77	272.03
102.41	78.23	84.54	265.18
98.43	81.33	94.60	274.36
107.31	79.13	89.69	276.13
105.57	78.55	81.85	265.97
92.93	73.20	83.53	249.66
95.15	74.13	84.11	253.39
96.00	73.98	81.82	251.80
112.69	80.55	88.20	281.44
105.37	78.33	85.12	268.82
100.84	75.46	87.27	263.57
99.64	73.63	88.15	261.42
104.26	78.68	83.59	266.53
99.33	77.55	91.48	268.36
102.95	81.04	92.33	276.32
99.61	76.76	82.32	258.69
104.65	81.66	87.85	274.16
108.54	78.88	85.05	272.47
101.85	73.91	82.33	258.09
102.03	80.70	92.63	275.36
95.57	73.66	85.54	254.77
107.83	74.06	83.75	265.64
102.47	75.70	89.45	267.62
98.77	75.38	82.00	256.15
99.50	77.73	85.17	262.40
103.55	75.73	87.23	266.51
103.78	76.16	81.66	261.60
99.44	74.80	92.85	267.09
99.96	70.25	87.90	258.11
105.80	79.48	88.20	273.48
mean	mean	mean	mean
101.92	76.95	86.70	265.57
std	std	std	std
4.26	2.87	3.72	7.79
std error	std error	std error	std error
.78	.52	.68m	1.46

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(2) **Trip Rate : 65% of capacity at Lock 22**
Distance : actual distance

lock 22	lock 24	lock 25	Total System
107.66	86.40	95.05	289.11
106.99	87.82	88.41	283.22
100.27	86.84	95.58	282.69
109.89	83.77	95.87	289.53
109.09	84.56	86.43	280.08
96.74	80.11	86.60	263.45
99.69	79.39	86.38	265.46
104.77	81.69	85.42	271.88
117.64	85.67	90.63	293.94
114.31	82.70	90.12	287.13
108.33	84.24	93.06	285.63
100.64	78.20	94.70	273.54
108.35	86.76	88.04	283.15
104.71	82.13	97.18	284.02
110.36	87.77	97.02	295.15
102.45	82.86	86.64	271.95
108.76	86.45	91.21	286.42
111.72	84.80	89.80	286.32
110.77	79.31	86.97	277.05
107.77	86.94	97.35	292.06
102.31	80.22	88.74	271.27
110.16	82.67	88.02	280.85
108.29	79.57	92.28	280.14
104.50	80.07	84.73	269.30
102.47	85.36	91.43	279.26
108.02	81.39	89.92	279.33
105.30	83.56	86.78	275.64
103.63	82.54	94.22	280.39
106.04	79.48	90.69	276.21
112.45	85.41	91.86	289.72
mean	mean	mean	mean
106.80	83.29	90.70	280.80
std	std	std	std
4.53	2.84	3.75	8.00
std error	std error	std error	std error
.83	.52	.68	1.46

* mean : average value
* std : standard deviation
* std error : standard error of the mean

(3) **Trip Rate : 65% of capacity at Lock 22**
Distance : 10 x actual distance

lock 22	lock 24	lock 25	Total System
113.22	98.70	101.51	313.43
115.20	101.56	98.61	315.37
112.81	100.90	101.65	315.36
115.57	99.01	96.56	311.14
117.15	93.41	97.55	308.11
109.33	87.53	89.43	286.29
106.72	91.65	96.01	294.38
114.10	94.01	99.14	307.25
119.09	101.84	100.53	321.46
115.97	94.57	90.79	301.33
111.26	101.46	98.64	311.36
112.23	94.97	96.89	304.09
112.96	96.76	96.60	306.32
113.06	98.41	100.08	311.55
114.49	97.13	100.98	312.60
113.45	93.29	92.42	299.16
114.10	97.33	97.55	308.98
117.95	94.17	95.00	307.12
116.48	90.83	93.20	300.51
114.06	98.57	104.96	317.59
107.50	93.95	92.78	294.23
122.74	93.51	94.47	310.72
114.64	92.44	92.57	299.65
116.43	92.99	93.86	303.28
114.01	94.98	98.47	307.46
111.48	94.16	93.98	299.62
119.39	96.38	91.61	307.38
108.15	91.59	96.31	296.05
110.07	91.23	102.24	303.54
112.51	100.16	97.79	310.46
mean	mean	mean	mean
113.87	95.58	96.74	306.19
std	std	std	std
3.50	3.58	3.70	7.64
std error	std error	std error	std error
.64	.65	.68	1.40

* mean : average value
* std : standard deviation
* std error : standard error of the mean

(4) **Trip Rate : 65% of capacity at Lock 22**
Distance : 100 x actual distance

lock 22	lock 24	lock 25	Total System
111.32	100.56	97.05	308.93
121.00	99.27	95.53	315.80
107.25	98.85	101.28	307.38
110.73	98.31	98.30	307.34
110.01	97.04	94.43	301.48
105.38	90.20	94.44	290.02
106.47	95.20	94.51	296.18
113.19	102.20	90.98	306.37
114.25	103.27	100.03	317.55
111.36	93.67	96.79	301.82
113.17	95.36	93.36	301.89
107.96	91.69	99.23	298.88
117.39	100.53	95.54	313.46
106.42	94.45	96.33	297.20
116.45	96.30	94.65	307.40
107.05	96.93	90.08	294.06
110.69	101.84	96.91	309.44
114.41	96.43	96.88	307.72
108.43	94.73	91.71	294.87
110.08	98.66	100.29	309.03
101.67	92.78	95.26	289.71
112.57	97.72	97.39	307.68
116.05	96.77	95.05	307.87
106.62	91.92	90.99	289.53
105.40	97.46	94.21	297.07
110.10	95.40	94.48	299.98
114.68	94.09	94.00	302.77
114.00	95.36	99.10	308.46
107.36	96.64	97.11	301.11
109.12	97.81	98.54	305.47
mean	mean	mean	mean
110.69	96.71	95.82	303.22
std	std	std	std
4.19	3.10	2.77	7.28
std error	std error	std error	std error
.76	.57	.51	1.33

* mean : average value
* std : standard deviation
* std error : standard error of the mean

Appendix 4

Simulated average tow delays at 95% of capacity (30 replications)

(1) Trip Rate : 95% of capacity at Lock 22

Distance : 0.1 x actual distance

lock 22	lock 24	lock 25	Total System
624.45	477.81	390.64	1492.90
798.69	504.71	454.41	1757.81
642.50	466.00	475.44	1583.94
742.37	457.81	430.48	1630.66
697.38	493.31	377.38	1568.07
680.39	364.21	350.03	1394.63
574.04	395.44	355.05	1324.53
707.62	434.43	391.80	1533.85
800.64	514.25	476.69	1791.58
746.49	393.78	380.79	1521.06
720.63	462.17	396.95	1579.75
683.77	473.61	406.68	1564.06
726.91	529.83	387.80	1644.54
691.44	451.61	403.11	1546.16
882.73	569.67	395.74	1848.14
676.43	356.05	345.31	1377.79
712.74	458.29	409.66	1580.69
722.54	532.11	449.49	1704.14
778.27	391.68	364.57	1534.52
704.11	505.22	414.47	1623.80
702.10	412.68	521.97	1636.75
689.67	450.74	377.98	1518.39
752.52	416.58	406.23	1575.33
578.22	393.34	369.57	1341.13
716.64	470.73	389.50	1576.87
665.10	441.95	369.59	1476.64
751.57	426.31	387.25	1565.13
665.63	450.43	484.87	1600.93
948.51	442.39	417.41	1808.31
723.96	494.98	424.33	1643.27
mean	mean	mean	mean
716.94	454.40	406.84	1578.18
std	std	std	std
74.87	50.20	41.82	123.98
std error	std error	std error	std error
13.67	9.16	7.63	22.64

* mean : average value

* std : standard deviation

* std error : standard error of the mean

(2) **Trip Rate : 95% of capacity at Lock 22**
Distance : actual distance

lock 22	lock 24	lock 25	Total System
652.45	497.68	414.84	1564.97
828.91	556.82	478.58	1864.31
662.96	502.78	510.04	1675.78
775.40	485.59	454.98	1715.97
713.71	540.17	397.43	1651.31
712.20	384.52	369.34	1466.06
600.94	426.32	391.44	1418.70
727.58	456.33	416.76	1600.67
826.92	544.43	504.30	1875.65
762.34	423.25	401.13	1586.72
760.19	485.30	416.70	1662.19
682.26	495.62	434.17	1612.05
749.33	539.84	410.70	1699.87
724.02	459.42	424.32	1607.76
888.01	584.55	417.93	1890.49
698.46	369.99	364.21	1432.66
729.31	478.99	430.29	1638.59
729.36	551.66	462.82	1743.84
805.84	412.82	383.62	1602.28
710.15	524.30	429.76	1664.21
708.16	426.27	552.00	1686.43
722.71	454.19	411.89	1588.79
788.51	430.27	423.94	1642.72
607.63	410.51	395.01	1413.15
733.09	489.25	414.32	1636.66
669.03	463.39	391.96	1524.38
764.75	444.20	408.79	1617.74
688.22	464.61	496.37	1649.20
949.37	458.05	445.02	1852.44
749.86	514.82	449.56	1714.24
mean	mean	mean	mean
737.39	475.86	430.07	1643.33
std	std	std	std
72.63	52.65	42.55	122.10
std error	std error	std error	std error
13.26	9.61	7.77	22.29

* mean : average value
* std : standard deviation
* std error : standard error of the mean

(3) Trip Rate : 95% of capacity at Lock 22
Distance : 10 x actual distance

lock 22	lock 24	lock 25	Total System
689.74	549.73	455.38	1694.85
881.01	605.02	538.54	2024.57
714.45	578.39	544.25	1837.09
888.27	534.86	529.60	1952.73
740.05	613.49	483.10	1836.64
745.49	445.53	436.75	1627.77
692.93	501.90	442.96	1637.79
773.00	509.13	464.96	1747.09
894.26	612.08	554.47	2060.81
873.23	509.32	479.61	1862.16
802.84	560.04	465.73	1828.61
708.46	593.98	467.39	1769.83
858.83	609.89	489.76	1958.48
776.64	524.55	470.0[illegible]	1771.25
916.39	692.51	502.8[illegible]	2111.70
769.38	474.91	397.7[illegible]	1642.07
802.14	522.19	504.96	1829.29
854.03	626.72	517.85	1998.60
912.33	477.49	436.15	1825.97
785.39	597.00	494.14	1876.53
738.26	474.66	630.86	1843.78
759.70	557.64	482.14	1799.48
841.29	496.15	461.81	1799.25
687.25	471.40	444.47	1603.12
835.45	517.09	478.99	1831.53
676.20	519.75	466.21	1662.16
869.71	519.01	488.54	1877.26
790.72	508.61	551.46	1850.79
1063.55	536.07	510.04	2109.66
827.72	576.00	494.62	1898.34
mean	mean	mean	mean
805.62	543.84	489.51	1838.97
std	std	std	std
85.66	55.99	44.64	134.66
std error	std error	std error	std error
15.64	10.22	8.15	24.59

* mean : average value
* std : standard deviation
* std error : standard error of the mean

(4) **Trip Rate : 95% of capacity at Lock 22**
Distance : 100 x actual distance

lock 22	lock 24	lock 25	Total System
977.50	676.46	496.40	2150.36
780.71	686.89	540.88	2008.48
824.33	600.16	668.05	2092.54
955.95	689.59	554.55	2200.09
934.33	665.27	488.65	2088.25
832.52	503.39	489.42	1825.33
764.69	651.04	487.71	1903.44
920.18	684.88	488.79	2093.85
857.02	796.42	604.53	2257.97
821.57	585.55	476.31	1883.43
887.42	606.37	596.00	2089.79
840.67	627.98	501.74	1970.39
851.37	661.30	539.57	2052.24
743.33	640.47	564.93	1948.73
1000.82	710.17	544.80	2255.79
693.99	558.20	455.70	1707.89
962.09	665.20	619.67	2246.96
895.74	786.02	542.88	2224.64
919.06	681.54	538.55	2139.15
862.81	723.42	618.15	2204.38
815.33	607.06	605.63	2028.02
837.13	586.65	498.82	1922.60
892.06	611.93	487.18	1991.17
804.13	522.79	534.20	1861.12
805.50	623.12	527.63	1956.25
795.79	667.42	499.07	1962.28
876.36	618.43	505.03	1999.82
671.39	618.12	520.79	1810.30
982.65	644.73	511.50	2138.88
752.81	714.29	534.24	2001.34
mean	mean	mean	mean
851.98	647.16	534.71	2033.85
std	std	std	std
82.19	64.52	49.78	140.93
std error	std error	std error	std error
15.01	11.78	9.09	25.73

* mean : average value
* std : standard deviation
* std error : standard error of the mean

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by Melody D.M. Dai[1], Paul Schonfeld[2] and George Antle[3]

Abstract

The congestion and variability of service times at locks significantly affect the cost and reliability of waterway transportation. This paper considers the effects of lock congestion levels and reliability on the operating cost of tows, assuming that tow operators have the opportunity to optimize speed in response to the delays they have already experienced and the delays they expect to encounter. The analysis method in this paper is useful for evaluating long-term consequences of lock improvements, as well as for optimizing speed from the viewpoint of operators.

This analysis method optimizes tow operations in two stages. The first stage finds the optimal speeds for each individual tow, re-optimizing the speed after every lock. The second stage determines the optimal allowed delivery times and associated optimal speeds based on the lock transit time distributions. The optimization is guided by a total cost objective function which includes penalties for late deliveries.

A four-lock section on the Ohio River is used for a case study in which various congestion levels and speed limits are tested. The resulting total cost functions are U-shaped with respect to the allowed delivery times. At given congestion levels, the optimal allowed delivery times and costs decrease as speed limits increase. The results also show how the optimal allowed delivery times and costs increase as congestion becomes severe.

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1. Introduction

The cost and reliability of waterway transportation is influenced considerably by congestion and by the variability of service times at locks. These locks enable or at least greatly facilitate navigation on waterways which may not be navigable otherwise. However, they may also constitute severe bottlenecks on those waterways as volumes increase.

The effects of congestion on mean travel times have been explored in various previous studies, including earlier parts of our work (e.g., Dai [2], Dai and Schonfeld [3]). The effects of variability in lock service times (and in consumption rates by end users) on inventory requirements and logistic costs was explored by Dai and Schonfeld [4]. This paper considers the effects of congestion levels and lock service time variability on the costs of operating barge tows, assuming that operators have the opportunity to optimize speed in response to expected delays. The analysis method presented here is intended for use in evaluating the long-term consequences of lock improvements, as well as for optimizing speeds in the short-term from the viewpoint of carriers. For the latter purpose, a more precise operating cost function than that provided so far would be required.

Although speed and reliability of travel times are less important for waterway transportation than for some other transportation modes, these variables significantly affect the costs of carriers and shippers. As speeds increase, power requirements and fuel costs increase far more than linearly but other costs such as labor and equipment depreciation per output unit (e.g. per ton mile) decrease. Higher speeds may also reduce the probability of late deliveries. Late deliveries impose significant costs on customers by increasing stock-outs as well as required inventories [4]. The costs associated with late deliveries may, at least sometimes, be passed along to tow operators in the form of penalties. As speeds increase, it is also possible to reduce the time allowed for deliveries, which presumably includes a scheduled travel time based on expected speeds and delays and a safety factor based on the variances of total travel time, including delays. Alternatively, for a given allowed delivery time, increased speed reduces the probability of late deliveries and penalties. Therefore, the optimal speeds and optimal allowed delivery times should be compromises among the costs of fuel, fleet and penalty costs. A total cost function formulated below is used to optimize tow speeds and allowed delivery times under changing traffic conditions.

The complete delivery time includes tow travel times and lock transit times (including lock service times and waiting times). In this study, the lock transit times are treated as probability distributions derived from a waterway simulation model [2] which has been validated against PMS (lock Performance Monitoring System) [6] data. Thus, each tow experiences a different transit time even at same lock. Such an assumption, which increases somewhat the complexity of this study, is more realistic than using only the average lock transit times. In addition, it allows us to study the relations among penalty costs, transit times, speeds and allowed delivery times which affect system reliability.

A case study of a four lock section on the Ohio River is presented below. Various volume/capacity (V/C) ratios and speed limits are tested. The total cost functions are U-shaped functions of the allowed delivery times. The results show how various volume to capacity (V/C) ratios and speed limits influence the optimal allowed delivery times and operating speeds.

2. Literature Review

Several analytic models and simulation models are available for estimating lock transit times. The characteristics of these models are discussed in this section.

2.1 Lock Delay Models

Two models based on the application of queuing theory have been found for estimating lock delays. DeSalvo and Lave modeled lock operation as an M/M/1 (Poisson arrivals/ exponential service times/1 server) queuing station [5]. Wilson modified this model as an M/G/1 (Poisson arrivals/General service times/1 server) queue [9]. Thus, the difference between these two models is the service time distribution. DeSalvo and Lave assumed that service times were exponentially distributed while Wilson relaxed this assumption by using generally distributed service times, which are more realistic. However, both models are limited to Poisson distributed arrivals, which may not be realistic even if locks are isolated and are quite unreasonable for closely spaced locks. In addition, the delays in both models were only analyzed for independent single-chamber locks without stalls (failures). In waterways, many locks may have two dissimilar chambers in parallel and the delays at adjacent locks may be highly related. Moreover, the stall occurrences interrupt lock operations and thus increase delays. Therefore, it is desirable to use a model which can analyze lock delays for entire interrelated series of locks and predict the effects of stalls and dissimilar parallel chambers.

2.2 Waterway Simulation Models

Howe [7] developed a system simulation model for analyzing lock delays and tow travel times. The service times in this model were based on empirically-determined frequency distributions. To avoid some troublesome problems and errors associated with the requirement to balance long-run flows in Howe's model, Carroll and Bronzini [1] developed another waterway system simulation model. Both models simulate waterway operations in detail but require considerable amounts of data and computer time, which limit their applicability for problems with large networks and numerous combinations of improvement alternatives. Both models assumed Poisson distributions for tow trip generation, which is not always realistic. More importantly for reliability analyses, neither of these models explicitly accounts for stalls, which are very different in frequency and duration from other events and affect overall transit time reliability.

Hence a waterway simulation model that explicitly accounts for stalls is desirable for evaluating the impacts of waterway reliability.

3. Simulation Model

A simulation model developed for related waterway studies (Dai [2], Dai and Schonfeld [3]) is used in this work. It may be used to determine the relations among delays, tow trips, distributions of generated tow trips, lock operations, lock service time distributions, travel times, as well as coal consumption and inventories at power plants supplied by tows. This simulation model, which is developed on the basis of PMS (lock Performance Monitoring System) data [6], can take into account stochastic effects such as stalls, randomly distributed arrivals and service times as well as seasonal variations.

It is a microscopic, event-scanning simulation model. It traces the movement of each individual tow and records its characteristics. It can handle any distributions for trip generation, travel speeds, lock service times and tow sizes. These distributions can be specified for each interval in tables or by standard statistical distributions. Currently, travel speeds are assumed to be normally distributed, while general distributions based on empirical observations are used for other input variables. Tows are allowed to overtake other tows. A FIFO (First-In-First-Out) service discipline is currently employed. The model simulates two-way traffic through common servers and accounts for stalls.

This simulation model is programmed in Fortran-77, which allows us to simulate relatively complex operations. The size of waterway systems that can be modeled is only limited by the computer capacity and the storage capacity of the Fortran compiler or linker. The simulation model has been developed with "*dynamic dimensioning*" to the degree allowed by the computer system available. Parameter statements are used so that the dimensions, and hence capacities, of the model components may readily be modified. This allows the maximum flexibility of waterway system representation and the most efficient computer utilization. Thus, the dynamic dimensioning programming technique allows flexibility in the number of locks, chambers, cuts, waterway links, tows, utility plants, origin-destination (O-D) pairs and simulation time periods.

Detailed descriptions and validation results for this simulation model are provided in Dai [2].

4. Methodology

4.1 Background

For this analysis it may be assumed that operators dispatch tows to meet certain delivery deadlines. If tows fail to meet their deadlines, penalties must be paid for the excess time. However, if operators try to avoid penalties by increasing their speeds, they incur higher fuel costs, which increase disproportionately with speed. Also, when speeds increase, the total delivery times decrease, thereby reducing fleet costs (i.e., equipment depreciation and labor). Thus, the operating speeds must be optimized through trade-offs among fuel costs, penalty costs, and fleet costs.

In this study, it is assumed that tow boats always try to operate at the most economic speeds to minimize the total costs. However, since lock transit times are somewhat uncertain, the speed for the remainder of a trip should be re-optimized after passing each lock, i.e., after that part of the uncertainty has been resolved. The new speeds would be based on the scheduled delivery times, expected remaining total lock transit times (including waiting times in queues and service times at locks), the remaining travel distance, unit fuel costs, unit penalty costs, and unit fleet costs. Sometimes, however, the most economic speeds may not be attainable due to physical or regulatory constraints.

This study assumed that tows with the same origin and destination have the same maximum allowed delivery times. The tow operator may optimize these by deciding how far in advance of the delivery deadline to dispatch a tow, i.e., how much travel time should be pre-scheduled. It is noted that the fleet size and the required delivery frequency affect the maximum allowed delivery times. When the delivery frequency remains constant, a larger fleet can allow longer delivery times. When the fleet size remains constant, reduced delivery frequencies allow longer delivery times. As allowed delivery times increase, the probability of missed delivery deadlines decreases and optimal tow operating speeds may also decrease. Therefore, long allowed delivery times require large fleet costs, but small fuel costs and penalty costs. This study aims to help carriers determine the optimal allowed delivery times and the optimal speeds between locks. The optimization is conducted in two stages. The first stage finds the optimal speeds for each individual tow, re-optimizing the speed after every lock. The second stage determines the optimal allowed delivery times for all tows serving a given origin destination pair. The optimal speeds and optimal allowed delivery times are determined based on the total cost function.

4.2 Optimal Speeds

The optimal speeds are determined for each individual tow boat after passing each lock since different tow boats may experience different lock transit times. The total cost function for optimizing speeds of the remaining delivery includes three components: (1) fuel cost, (2) penalty cost, and (3) fleet cost.

$$C_t = C_{fu} + C_p + C_{fl} \tag{1}$$

where

C_t : total cost for the remaining delivery distance

C_{fu} : fuel cost for the remaining delivery distance

C_p : penalty cost for the remaining delivery distance

C_{fl} : fleet cost for the remaining delivery distance

The fuel cost increases as the speed increases. In this study, it is assumed that the unit fuel cost is proportional to the square of the speed. The fuel cost for the remaining delivery distance can be represented as:

$$C_{fu} = c_{fu}(V)\, D_R \qquad (2)$$

where

$c_{fu}(V)$: unit fuel cost when a tow is operating at speed V, \$/tow-mi

V : speed, mi/hr

D_R : remaining travel distance, mi

The unit fuel cost is assumed to be a constant k multiplied by the square of the speed:

$$c_{fu}(V) = k\, V^2 \qquad (3)$$

The constant k is estimated from cost values in the Reebie [8] study of barge operating costs. An improved cost function may easily be substituted in this model when it becomes available.

The penalty cost is charged when the delivery time exceeds the deadline, and thus can be represented as:

$$C_P = c_p\, B\, \max[(D_R/V - T_R),\ 0] \qquad (4)$$

where

c_p : unit penalty cost, \$/barge-hr

B : tow size, barges/tow

T_R : expected remaining travel time, hr

$$T_R = T_A - T_U - T_{LR} \qquad (5)$$

T_A : allowed delivery time, hr

T_U : used delivery time, hr

T_{LR} : expected remaining lock transit time, hr

The fleet cost for the remaining delivery is assumed to be proportional to the remaining delivery times and can be represented as follows:

$$C_{fl} = c_{fl}\ B\ (D_R/V + T_{LR}) \qquad (6)$$

where

c_{fl} : unit fleet cost, \$/barge-hr

Therefore, Eq. 1 can be expanded as a constrained minimization problem:

$$\text{minimize} \quad C_t = k\ V^2\ D_R + c_p\ B\ \max[D_R/V - T_R, 0] + c_{fl}\ B\ (D_R/V + T_{LR}) \qquad (7)$$

$$\text{subject to} \quad V \leq V_{MAX} \qquad (8)$$

$$V \geq V_{MIN} \qquad (9)$$

where

V_{MAX} : maximum speed limit

V_{MIN} : minimum speed limit

The maximum speed limits may be based on mechanical limitations or policy constraints. For generality, minimum speed limits may also be imposed but that is usually unnecessary for waterways. Therefore, the minimum speed is assumed to be zero in this study.

To solve we must consider whether penalties (1) should be paid for the sake of more economical speeds or (2) should not be accepted. Assuming penalties are acceptable, without considering the constraints of speed limits, the optimal speed V_1 is obtained by setting the deriviative of C_t (Eq 7) equal to zero and solving for V:

$$V_1 = [(c_p + c_{fl})/(2\ k)]^{1/3} \qquad (10)$$

The above solution assumes some penalty is paid. However, it is necessary to check whether V_1 is less than the critical speed V_{CR}, which represents the boundary speed between paying or not paying penalties. Tows need not pay penalties when their speeds exceed V_{CR}. The critical speed V_{CR} can be represented as follows:

$$V_{CR} = D_B/T_R \tag{11}$$

If penalties are not paid, and without considering the constraints of speed limits, the optimal speed V_2 can be obtained by differentiating the total cost function (Eq. 7), setting it equal to zero and solving for V. Then the optimal speed is

$$V_2 = [c_0/(2\ k\ F_U)]^{1/3} \tag{12}$$

It is also necessary to check whether V_2 is greater than V_{CR}. In addition, both V_1 and V_2 need to satisfy the speed limits. Otherwise, tows operate at the nearest feasible speed limits.

4.3 Optimal Allowed Delivery Times

The optimal speeds derived in the previous section are for individual tows with a specific allowed delivery time. Different allowed delivery times require different optimal speeds, and hence, the associated total costs are different. This section discusses the model for finding the optimal allowed delivery times for tows moving between the same origins and destinations. It is noted that the costs in this model represent the optimal costs for each individual tow with a certain delivery deadline. The total cost function can be represented as follows:

$$C_T = C_{FU} + C_P + C_{FL} \tag{13}$$

where

C_T : total cost for tows connecting a given origin-destination pair

C_{FU} : fuel cost for tows connecting a given origin-destination pair

C_P : penalty cost for tows connecting a given origin-destination pair

C_{FL} : fleet cost for tows connecting a given origin-destination pair

To estimate the total costs, it is necessary to know the operating speeds and delivery times. This study assumes that each individual tow may experience different lock transit time at the same lock, based on observed or simulated distributions. Therefore, the operating speeds and delivery times are different for each individual tow. This assumption increases the difficulty of estimating the total costs unless the movement of each tow is traced by simulation. To avoid the time-consuming simulation, this study splits tows into several groups. The tows in each group have similar delivery times. Then, optimal speeds and travel times are determined for each group. It is noted that the tows in same group may experience different lock transit times at the next lock. Therefore, after passing each lock, it is necessary to regroup the tows. Equivalently, the service time distribution is divided into n intervals (n is usually 10 here) and tows are re-distributed into those n intervals after each lock.

The total fuel costs for all tows in this service can be represented as

$$C_{FU} = \Sigma_i \Sigma_j c_{fu}(V_{ij}) M_{ij} \tag{14}$$

where

V_{ij} : average speed of Speed Group j in Segment i, mi/hr

M_{ij} : total barge miles for Speed Group j in Segment i, barge-mi/hr

$$M_{ij} = Q_{ij} F_L B L_i \tag{15}$$

Q_{ij} : number of tows in Speed Group j in Segment i, tows/mi

L_i : length of Segment i, mi

The penalty costs are charged when tows are late, resulting in total penalty costs C_P for the entire fleet of

$$C_P = c_p \Sigma_j T_{Dj} Q_{Ij} B \tag{16}$$

where

T_{Dj} : number of average late hours/tow for tows in Speed Group j

Q_{Ij} : number of tows/hr in Speed Group j in the last Segment I

The fleet operating costs C_{FL} are represented as follows:

$$C_{FL} = c_{fl} \Sigma_j Q_{Ij} B (T_A + T_{Dj}) \tag{17}$$

Therefore, the total costs in Eq. 13 can be expanded as follows:

$$C_T = \Sigma_i \Sigma_j c_{fu}(V_{ij}) Q_{ij} F_L B L_i + c_p \Sigma_j T_{Dj} Q_{Ij} B + c_{fl} \Sigma_j Q_{Ij} B (T_A + T_{Dj}) \tag{18}$$

The optimal allowed delivery time is that which minimizes the above total cost.

4.4 Lock Transit Times

To optimize speeds and allowed delivery times, we must consider lock transit times as well as travel times between locks. For the same allowed delivery times, if the lock transit times increase, the allowed travel times decrease, thus increasing expected penalties.

The lock transit times include two components: (1) lock service times and (2) waiting times. This model uses information on the distributions of lock transit times as well as the mean ("expected") lock transit times. The expected lock transit times are useful when the tow operators want to adjust speeds for the remaining distance. At the time operators adjust their speeds, the remaining lock transit times are still uncertain. They can adjust their speeds based on estimated probable transit times, which are in turn based on prior data. The distributions of lock transit times are useful for estimating the lock transit time for each individual tow.

In this study the lock transit times are estimated by simulating a series of locks and recording the transit times for each tow. Therefore, the simulation results can provide the total transit times at each lock for each individual tow and may be used to estimate averages and probability distributions. To adjust speeds for the remaining distance, it is necessary to know the expected total transit times through all the remaining locks between the current locations and destinations.

The distribution of lock transit times at each lock is specified in table form for several groups (10 groups in this study). The average lock transit time and the probability table of lock transit times for each group are also obtained from the simulation.

5. Case Study

A four lock section of the Ohio River around the Gallipolis Lock was selected for a case study since that lock constitutes a relative bottleneck in the waterway capacity. Compared to the other three locks nearest to it (Belleville, Racine, and Greenup), Gallipolis is the oldest and its two chambers are the smallest.

The Stuart utility plant of the Dayton Power and Light Co., located between the Greenup and Meldahl locks, is chosen for this case study. It is 63.5 miles downstream from Greenup and 31.7 miles upstream from Meldahl.

There are five segments in this study. The segment characteristics are shown in Table 5-1 and the lock transit times are shown in Table 5-2.

Table 5-1 Segment Characteristics

Segment	From	To	Length (mi)
1	Origin	Belleville	21.1
2	Belleville	Racine	33.6
3	Racine	Gallipolis	41.7
4	Gallipolis	Greenup	61.8
5	Greenup	Destination	63.5

Table 5-2 Lock Transit Times

Lock	Lock Transit Time (hr) V/C=0.45[1] AVG	STD	V/C=0.80[1] AVG	STD	V/C=0.95[1] AVG	STD
Belleville	0.83	0.37	0.92	0.50	0.96	0.54
Racine	1.10	0.98	1.57	2.09	1.75	2.49
Gallipolis	3.92	3.66	11.83	11.05	48.84	39.52
Greenup	0.79	0.36	0.92	0.52	0.97	0.58

1. The V/C ratio is measured at Gallipolis Lock

The baseline values for the other variables are listed in Table 5-3.

Table 5-3 List of Variables and Baseline Values

Variable	Description	Value
B	average tow size, barges/tow	8.04
c_{fl}	unit fleet cost, \$/barge-hr	28.56[1]
c_p	unit penalty cost, \$/barge-hr	50
k	parameter for unit fuel cost	0.02296[1]
V_{MIN}	minimum speed limit, mi/hr	0

1. Source [8]

The case study finds the optimal speeds and optimal allowed delivery times for different combinations of speed limits and volume to capacity ratios (V/C). The associated costs also are provided in this case study. Tables 5-4 lists the optimal costs for various allowed delivery times and speed limits when the volume/capacity (V/C) ratio at the critical Gallipolis lock is 0.80.

Table 5-5 shows the optimal costs for various V/C ratios at Gallipolis Lock. The optimal costs for various standard deviation of lock service times are listed in Table 5-6. In Table 5-6, the ratio of standard deviation of lock service times is measured against the baseline case when V/C ratio is 0.80 at Gallipolis Lock.

Figure 5-1 shows the relation between the total costs and the allowed delivery times for different speed limits when the V/C ratio is 0.80. The total costs are U-shaped curves. They first decrease and then increase as the allowed delivery times increase. In fact, when the allowed delivery times are small, tows have higher speeds and fuel costs and, possibly, higher penalty costs although fleet costs may decrease. At first, as allowed delivery times increase, the fuel and penalty savings outweigh more than the extra fleet costs. However, beyond a certain range, the marginal benefit decreases and is outweighed by the increased fleet costs associated with the increased allowed delivery times. This graph also shows that as the speed limits increase, the optimal allowed delivery times and associated total costs decrease until reaching a limit. When speed limits are low, tows may have to operate below their most economic speeds. Therefore, relaxation of speed limits may sometimes reduce total costs.

Figure 5-2 shows the optimal total costs for different V/C ratios. In this graph, the optimal total costs increase as the V/C ratios increase. In Table 5-2, as V/C ratios increase, the congestion becomes severe and the mean and standard deviation of lock transit times increase. Especially when the V/C ratio is 0.95 at the Gallipolis Lock, the mean and standard deviation of lock transit times increase substantially. The standard deviation of lock transit times is an indicator of lock reliability. The higher the standard deviation of lock transit times, the less reliable the lock is. Therefore, as V/C ratios increase, the variance as well as the mean of the transit times increase, thus reducing service reliability and increasing delivery times and costs.

Figure 5-3 shows the optimal total costs for various standard deviations of lock service times. In it, the optimal total costs increase as the standard deviations, and thus the unreliability, of lock service times increase. As the standard deviation of lock service times increases, the overall travel time reliability is reduced, and thus, the optimal total cost increases.

Table 5-4 Optimal Costs for Different Allowed Delivery Times and Speed Limits (V/C=0.80)

T_A (hr)	V_{MAX} (mi/hr)	C_{FU} ($/tow)	C_P ($/tow)	C_{FL} ($/tow)	C_T ($/tow)
24	6.3	1988.57	10740.64	11645.99	24375.19
36	6.3	1988.57	5916.64	11645.99	19551.19
48	6.3	1988.57	2161.96	12256.79	16407.31
60	6.3	1988.57	788.72	14227.86	17005.15
72	6.3	1988.57	248.84	16674.95	18912.35
84	6.3	1988.57	.00	19288.28	21276.85
96	6.3	1988.57	.00	22043.75	24032.32
108	6.3	1988.57	.00	24799.22	26787.79
120	6.3	1988.57	.00	27554.69	29543.25
24	8.3	3532.40	7176.10	9609.93	20318.43
36	8.3	3467.30	2849.38	9893.97	16210.64
48	8.3	3237.95	1086.20	11642.31	15966.47
60	8.3	3212.40	398.78	14005.13	17616.31
72	8.3	3193.23	.00	16532.81	19726.04
84	8.3	3193.23	.00	19288.28	22481.51
96	8.3	3193.23	.00	22043.75	25236.98
108	8.3	3193.23	.00	24799.22	27992.45
120	8.3	3193.23	.00	27554.69	30747.92
24	10.4	5527.33	5037.46	8388.33	18953.13
36	10.4	4754.28	1809.40	9299.94	15863.62
48	10.4	3466.02	842.99	11503.39	15812.41
60	10.4	3325.15	278.18	13936.24	17539.58
72	10.4	3193.23	.00	16532.81	19726.04
84	10.4	3193.23	.00	19288.28	22481.51
96	10.4	3193.23	.00	22043.75	25236.98
108	10.4	3193.23	.00	24799.22	27992.45
120	10.4	3193.23	.00	27554.69	30747.92
24	12.5	6226.45	4538.98	8103.60	18869.04
36	12.5	5000.28	1663.88	9216.81	15880.97
48	12.5	3548.95	784.30	11469.87	15803.12
60	12.5	3366.61	248.44	13919.25	17534.30
72	12.5	3193.23	.00	16532.81	19726.04
84	12.5	3193.23	.00	19288.28	22481.51
96	12.5	3193.23	.00	22043.75	25236.98
108	12.5	3193.23	.00	24799.22	27992.45
120	12.5	3193.23	.00	27554.69	30747.92
24	14.6	6226.45	4538.98	8103.60	18869.04
36	14.6	5000.28	1663.88	9216.81	15880.97
48	14.6	3548.95	784.30	11469.87	15803.12
60	14.6	3366.61	248.44	13919.25	17534.30
72	14.6	3193.23	.00	16532.81	19726.04
84	14.6	3193.23	.00	19288.28	22481.51
96	14.6	3193.23	.00	22043.75	25236.98
108	14.6	3193.23	.00	24799.22	27992.45
120	14.6	3193.23	.00	27554.69	30747.92

Table 5-5 Optimal Costs for Different V/C Ratios

V/C	C_T ($/tow)
0.45	11902.66
0.80	15803.12
0.95	31737.85

Table 5-6 Optimal Costs for Different Standard Deviations of Lock Service Times

Ratio of S.D.[1]	C_T ($/tow)
0.95	15741.94
0.98	15760.91
1.00	15803.12
1.03	15816.03
1.07	15874.33
1.09	15942.45

1. Ratio of S.D.: Ratio of standard deviations of service times to the baseline case (V/C=0.80)

6. Conclusions

A methodology is developed for analyzing how tow operating costs are affected by operator-controlled variables such as speeds and allowed delivery times as well as exogenous factors such as congestion levels, lock service time means and variances, fuel cost rates and other operating cost factors. Speeds and allowed delivery times, which are important decision factors for carriers, may be optimized with this approach. The total cost function for optimizing speeds and allowed delivery times includes fuel costs, penalty costs and fleet costs.

The optimization of speeds and allowed delivery times is conducted in two stages. The first stage finds the optimal speeds associated with different allowed delivery times and speed limits. The optimal speeds are determined for each individual tow after passage through each lock. The second stage determines the optimal allowed delivery times based on the optimal speeds obtained in Stage 1.

A four-lock section constrained by the Gallipolis Lock was selected for a case study. The results show that the total costs are U-shaped functions of the allowed delivery times. As the speed limits increase, the optimal allowed delivery times and costs decrease toward asymptotic limits. In addition, as V/C ratios increase, the optimal allowed delivery times and costs also increase.

This methodology is useful for evaluating tow operations with uncertain lock transit times. It can be improved in the following aspects:

1. Improved cost functions should be developed.
2. Penalty costs should be related to actual inventory or stock-out costs incurred by shippers.
3. The effects of service reliability on demand should be analyzed.

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Appendix - List of Variables

B : tow size, barges/tow

C_{FL} : fleet cost for tows connecting a given origin-destination pair

C_{FU} : fuel cost for tows connecting a given origin-destination pair

C_P : penalty cost for tows connecting a given origin-destination pair

C_T : total cost for tows connecting a given origin-destination pair

C_{fl} : fleet cost for the remaining delivery distance

C_{fu} : fuel cost for the remaining delivery distance

C_p : penalty cost for the remaining delivery distance

C_t : total cost for the remaining delivery distance

c_{fl} : unit fleet cost, \$/barge-hr

$c_{fu}(V)$: unit fuel cost when a tow is operating at speed V and fully loaded, \$/barge-mi

c_p : unit penalty cost, \$/barge-hr

D_R : remaining distance to travel, mi

L_i : length of Segment i, mi

M_{ij} : total barge mile for Speed Group j in Segment i, barge-mi/hr

Q_{Ij} : number of tows/hr in Speed Group j in the last Segment I

Q_{ij} : number of tows in Speed Group j in Segment i, tows/mi

T_A : allowed delivery time, hr

T_{Dj} : number of average late hours/tow for tows in Speed Group j

T_{LR} : expected remaining lock transit time, hr

T_R : expected remaining travel time, hr

T_U : expended delivery time, hr

V : speed, mi/hr

V_{CR} : boundary speed between paying or not paying penalties

V_{MAX} : maximum speed limit

V_{MIN} : minimum speed limit

V_{ij} : average speed of Speed Group j in Segment i, mi/hr

V_1 : unconstrained optimal speeds with penalties

V_2 : unconstrained optimal speeds without penalties

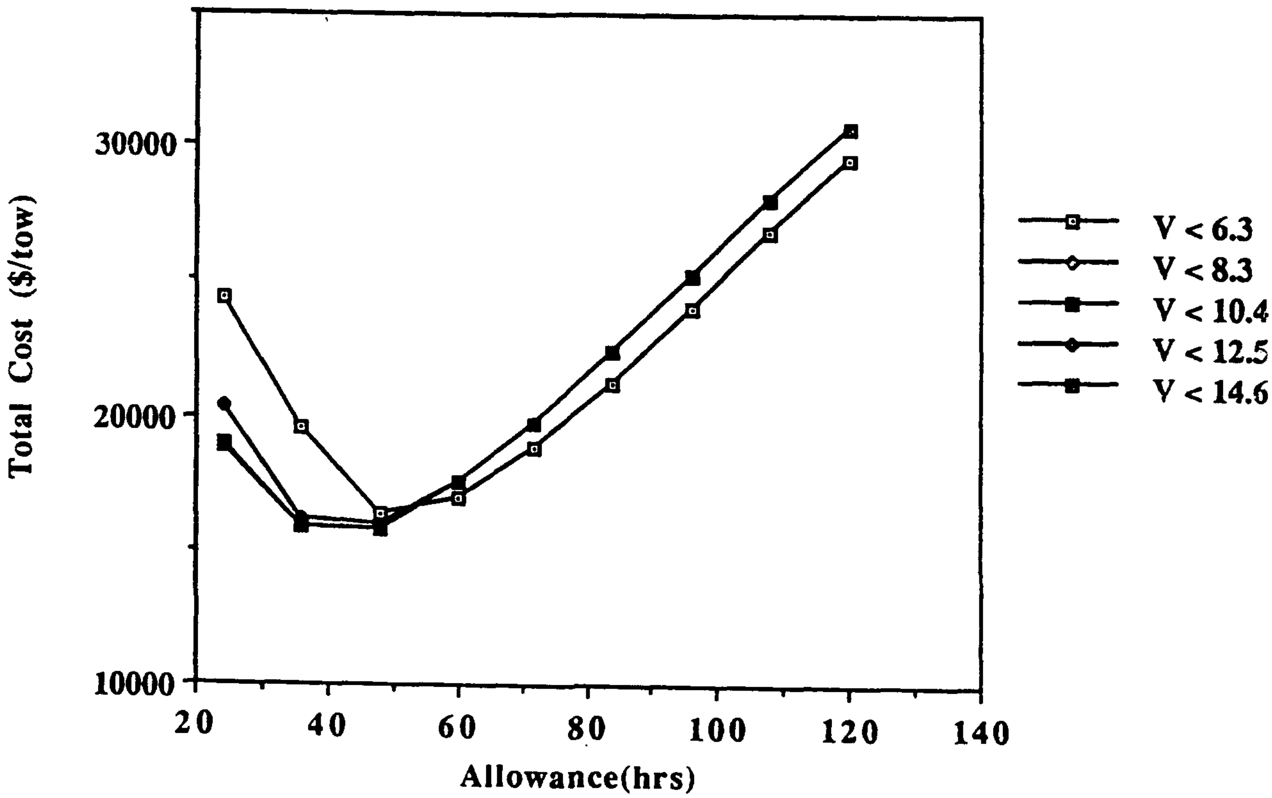

Figure 5-1 Total Costs for Various Speed Limits (V/C=0.80)

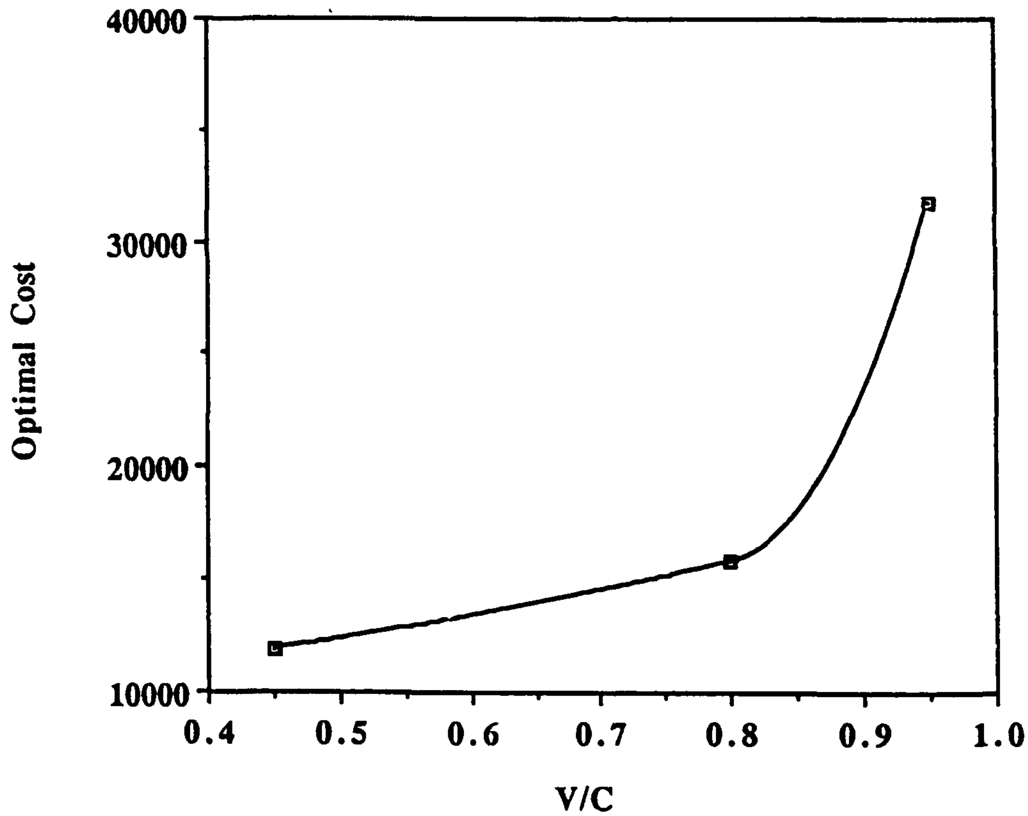

Figure 5-2 Optimal Costs for Various V/C Ratios

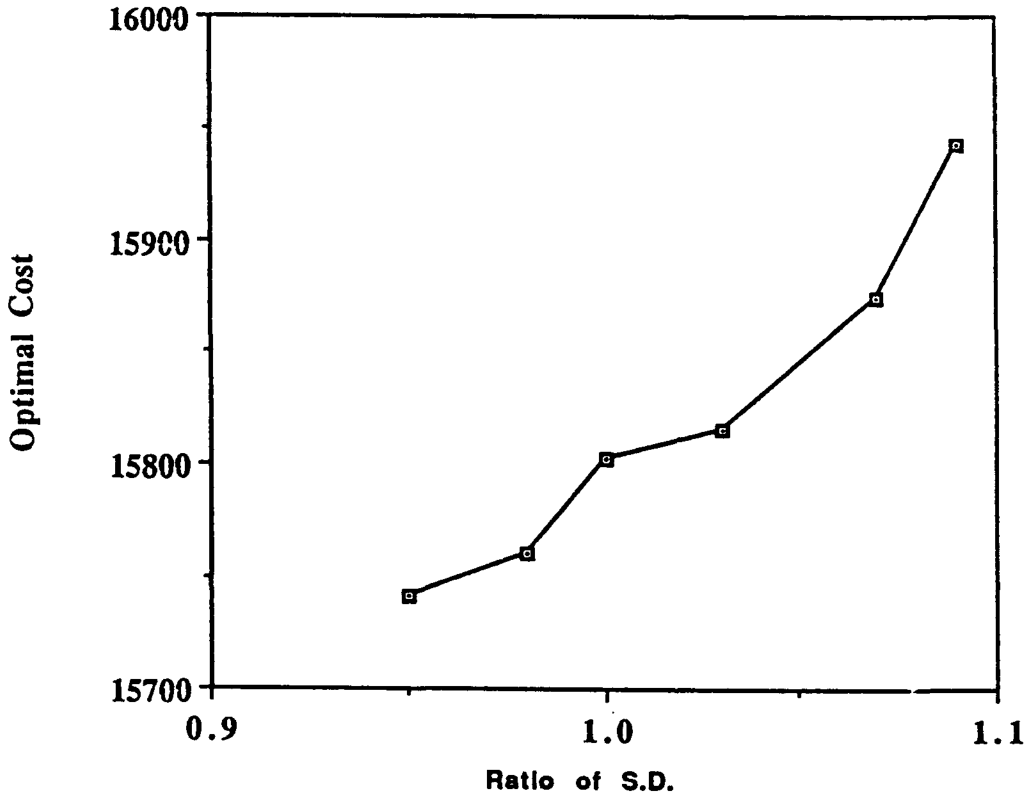

Figure 5-3 Optimal Costs for Various Standard Deviations of Lock Service Times

PRIORITIZING AND SCHEDULING INTERDEPENDENT LOCK IMPROVEMENT PROJECTS

by

David Martinelli and Paul Schonfeld

Prioritizing and Scheduling Interdependent Lock Improvement Projects

David Martinelli[1], AM ASCE and Paul Schonfeld[2], AM ASCE

Abstract

Current methods of capital budgeting are quite satisfactory for analyzing mutually exclusive projects and reasonably satisfactory for independent projects. However, there is a void in existing methodologies in prioritizing and scheduling projects that are interdependent. This is because project evaluation, sequencing, and scheduling must be performed simultaneously in order to yield an exact solution. The benefits (or cost reductions) associated with navigational lock improvements are interdependent, i.e. the improvement benefits of a given lock are affected by the acceptance or rejection of other lock improvement projects. Therefore, the problem is beyond the scope of conventional solution techniques. In this paper, we develop a methodology for obtaining an optimal or near optimal sequence and schedule of projects subject to a limited budget. Given a model to compute capital and delay costs for interdependent lock improvements (evaluation), the algorithm presented in this paper may be used to search the solution space of possible project permutations of sequences (prioritization) as well as obtain the start times for projects subject to a budget constraint (scheduling). A validation of the algorithm is also provided yielding promising results.

Keywords

capital budgeting, project prioritization, project scheduling, interdependent projects

Summary

An algorithm for sequencing and scheduling interdependent lock improvement projects subject to a limited budget is developed. The problem representation and solution method overcome many of the difficulties with conventional approaches. A validation of the algorithm is also provided.

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1. Introduction

Capital budgeting is the process of determining which investments or "projects" will be funded and pursued in order to meet prespecified goals and objectives over a planning horizon. A set of projects to be implemented at specific times constitute a capital investment program. In order to assist decision makers in funding decisions, capital budgeting models have been proposed for use in planning based on various quantifiable criteria. Techniques such as present worth economics, risk analysis, "what if" financial models, and mathematical programming have all been employed in capital budgeting. The complete capital budgeting process may be divided into three components: project evaluation, project selection, and project scheduling.

The *project evaluation* phase involves quantitatively assessing the benefits and costs of each project under consideration for each period of the planning horizon. The *project sequencing* phase uses measures of effectiveness (MOEs), agency or firm priorities, budget, and other factors to determine the relative priority of projects. The *project scheduling* phase assigns a start time to each project. While this phase is conceptually simple, budgets and other constraints may impose delays on project start times.

This paper reports the development of a method that accounts for system effects in inland waterway improvement projects. Current methods of capital budgeting are quite satisfactory for analyzing mutually exclusive projects and reasonably satisfactory for independent projects. However, there is an obvious void in analyzing projects that are interdependent. Interdependencies exist whenever the benefits or costs of any one project may depend on the acceptance of one or more other projects. It seems that overcoming this void requires three tasks: 1) the development of a framework whereby application-specific evaluation functions may be formulated for aggregating benefits and costs among interdependent projects, 2) the development of a technique whereby the numerous permutations of possible programs may be represented and searched, and 3) the determination of efficient project implementation schedules. This paper focuses on Tasks 2 and 3, while Task 1 is discussed in Martinelli and Schonfeld (1992).

The standard analysis techniques for capital investment projects do not deal with the interdependencies among available projects. For this reason, it is common practice to reduce all problems to either independent or mutually exclusive sets of projects. This is usually done by amalgamating those projects with strong interdependencies and ignoring any remaining interdependencies. The extent to which this practice leads to good decisions has been obscured by the lack of analysis techniques capable of adequately representing interdependencies in specific instances. The objective of this paper is to introduce a heuristic technique for the sequencing and scheduling of interdependent lock improvement projects. While the context for the development for the technique is lock improvements, it is likely that the technique could be applied to other network-based capacity expansion programs.

2. Sequencing and Scheduling for Lock Improvements

The National Waterways Study identified a need for substantial investment in the waterway infrastructure (USACE 1987). This need stems from 1) waterway traffic projections that approach or exceed the capacity of some existing facilities and 2) the age and physical deterioration of facilities. Currently there are about 100 locks that have exceeded their 50 year design life. Experience with aging locks indicates that lock closures or stalls and subsequent navigational delays can be expected to increase as locks age. Also, aging locks tend to have substantially longer tow processing times. Stalls and high processing times can result in increased shipping costs, delayed shipments, loss of cargo, higher logistics costs, and other adverse effects.

By far the most significant of these benefits is the reduction in trip delays associated with expanded capacity resulting from reconstruction. However, the delays at a given lock may depend significantly on conditions at various other locks, thus introducing the difficulties associated with interdependent project sequencing and scheduling (Martinelli 1991).

3. Problems with Interdependent Sequencing and Scheduling

Explicitly, the project sequencing process chooses a subset of n investment projects from a set of N desirable projects in the most desirable order. The problem confronting the decision analyst is to choose from among the N! possible permutations of project sets, the one which yields the maximum return. One possible method of selecting a set of projects might be to choose the highest payoff set out of a complete enumeration of sets that satisfy the budget and other constraints. However, as a practical matter, complete enumeration becomes infeasible as a method of finding the optimal sequence. If one is to consider, by complete enumeration, all of the possible sequences of 30 projects, then about 2.6×10^{32} alternative sequences must be examined. Clearly, it becomes prohibitively expensive to select and/or sequence interdependent projects through complete enumeration of alternatives.

If interactions among projects are assumed to be limited to pairwise interactions, the problem may be formulated as a 0-1 integer programming problem where the objective is to maximize the total net present value (TNPV) subject to a budget constraint for each period. The decision variables y_{is} indicate the projects to be implemented and their appropriate start dates.

$$\max \quad TNPV = \sum_{i=1}^{N} \sum_{s=1}^{T} \left(NPV_{is}\, y_{is} + \sum_{j \neq i}^{T} \sum_{s=1}^{T} \sum_{v=1}^{T} d_{ijsv}\, y_{is}\, y_{jv} \right) \qquad \text{Eq. 1}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{s=1}^{t} C_{ist}\, y_{is} + D_t - D_{t-1}(1+I) = B_t \qquad t=1..T \qquad \text{Eq. 2}$$

$$y_{is} \in (0,1) \qquad s=1...T,\ i=1...N \qquad \text{Eq. 3}$$

$$\sum_{s=1}^{T} y_{is} \leq 1 \qquad i=1...N \qquad \text{Eq. 4}$$

$$D_0 = 0 \qquad \text{Eq. 5}$$

$$D_t \geq 0 \qquad t=1...T \qquad \text{Eq. 6}$$

In the above formulation, NPV_{is} is the net present value of project i with start time s, while d_{ijsv} represents the deviation (in present dollars) from linear addition in the net return from two interacting projects i and j with start times s and v, respectively. This deviation may be more explicitly stated as difference between the deviation in benefits and the deviation in costs

$$d_{ijsv} = b_{ijsv} - c_{ijsv} \qquad \text{Eq. 7}$$

where b_{ijsv} and c_{ijsv} are the is the deviation in present value of benefits and costs, respectively, for two interacting projects.

In the above formulation, C_{ist} is the required expenditure in period t, for project i, starting in period s, D_t is the unspent budget in period t, I is the interest rate, and B_t is the limit on expenditures in period t. N is the total number of all projects considered, while T is the number of time periods in the planning horizon. The variable D_t is included to allow unspent portions of the budget in each period to be "rolled over" into the budgets of succeeding periods. The decision variable y_{is} is 1 if project i is to start in period s, and 0 otherwise.

There are significant shortcomings with this formulation. First, only paired interactions are represented. Depending on the application, three, four, or more projects may be simultaneously dependent. Second, the number of integer variables is excessive. For example, a problem with 30 projects and a planning horizon of 50 years (time periods) could have 1,500 binary integer (y_{is}) decision variables. This same problem would also require approximately 2.25 million interaction coefficients (d_{ijsv}'s) as well as 75,000 cost parameters (C_{ist}'s). In general, the interaction variables are quite difficult to estimate. While many problems may be smaller than this example, mixed-integer programming packages have serious difficulties with problems of this size. Also, interdependencies are only considered to exist among those projects that are actually implemented. There is a need to formulate the problem in a manner that is not as computationally expensive and does not require excessive estimation of interaction parameters.

4. Methodology

4.1 Project Sequencing Procedure

The proposed approach for searching the solution space of possible project permutations represents the solution space in two dimensions and applies a heuristic search algorithm in selecting the preferred sequence. Given a system cost evaluation function for interdependent projects $g(X,Y)$, the selection and sequencing problem may be represented in two dimensional space. The function $g(X,Y)$ incorporates both benefit and cost factors into a generalized cost while accounting for project interdependencies where X is a vector of delay variables and Y represents a particular combination of projects.

Assuming that each set of projects may be viewed as a system generating a common time-dependent output, then a two dimensional representation is quite feasible. For the lock rehabilitation problem, the costs associated with a given combination of projects in a given time period t, may be written as

$$(SC)_t = C_i + g(X,Y)_t\, O_W \qquad \text{Eq. 8}$$

where C_i is the total capital cost of construction for project i. The term $g(X(\lambda),Y)_t$ represents the delay, and corresponds to the function(s) obtained from some interdependent evaluation, e.g. from a simulation model, while O_W is the opportunity cost of delay. Evaluating SC at different levels of output for a combination of projects Y, defines a curve with annual system costs SC_t on the vertical axis and output level, λ, on the horizontal axis. Repeating for different values of Y (i.e. different project combinations) produces a family of curves. By always choosing the lowest cost curve for any given output level λ, i.e. by choosing the "lower envelope" of the curves in Figure 1, a sequencing and scheduling decision path is defined. Because the output is assumed to be time dependent, the horizontal axis may also represent time periods, e.g. years. Output and time may be linked through a demand function, $\lambda(t)$.

Consider an example with interdependent projects A, B, and C. Figure 1 shows a family of system cost (SC) curves corresponding to the possible combinations of these three projects. Note that in general, combinations involving only one project are preferable (lower SC) for low levels of volume (thus earlier in the horizon stage), and become less preferable as volume increases. Under this representation, one combination is preferred to another at a given output level (or time period) if its corresponding curve lies above the other.[3]

In the example depicted in Figure 1, the selection and sequence of projects is dictated by the lower "envelope" fined by the curves. This lower envelope corresponds to the minimization of the time integral of the system cost for feasible expansion paths. Here, all three projects would be accepted if the volume level is expected to eventually exceed Q_2. We see also that the sequence of projects should be A, B, C; this is because Curve A lies below B and C, and AB lies below AC in the relevant regions. Project A is preferred up to volume level Q_1 at the same time Project B should be implemented since Curve AB falls below Curve A. At volume level Q_2, Project C should be added to A and B, thus implementing Combination ABC.

[3]Although the convex and monotonically increasing properties of the curves in Figure 1 are likely to occur for costs with a delay component, they are not a prerequisite for the methodology.

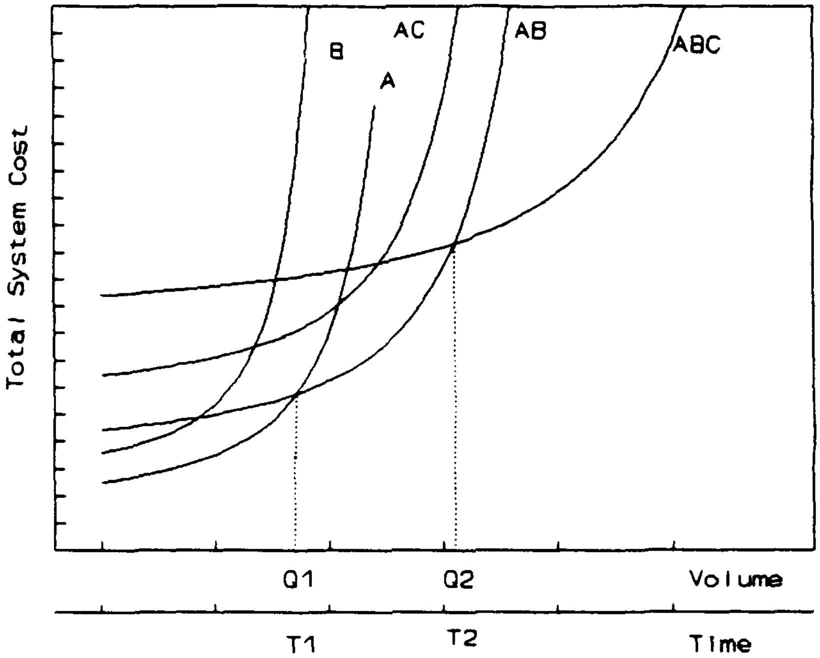

Figure 1 Plot of System Cost for Three Interdependent Projects (Case 1)

A second case involving the three projects A, B, and C is shown in Figure 2. Here, because Curve AB lies completely below Curve ABC, only Projects A and B are included in the program. The sequencing decision would be the same as in the previous example, with the intersection of Curves A and AB indicating that the implementation of Project B should be timed at t_1.

Unfortunately not all such families of curves can be interpreted as easily as Cases 1 and 2. Consider a third case shown in Figure 3 where Curves A and AB are unchanged but the others are different. Here, Curves AB and AC intersect each other before intersecting Curve ABC. It cannot be stated a priori weather Combination AB or AC should be selected on the expansion path between A and ABC. One would expect that if Area 1 is greater than Area 2, then Combination AB is preferred to AC and Project B should precede Project C on the expansion path. Areas 1 and 2 correspond to the difference savings when integrating over Paths

A-AB-ABC and A-AC-ABC, respectively.

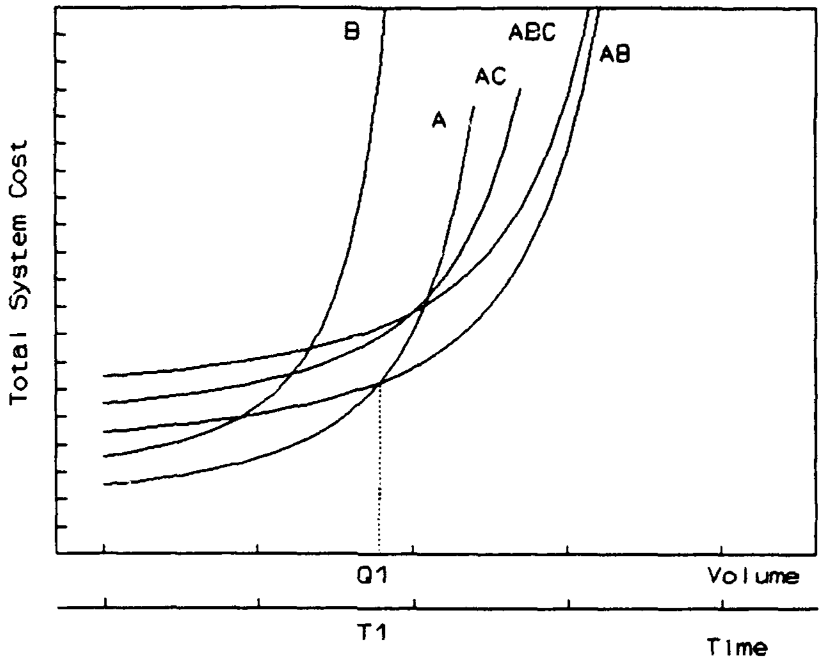

Figure 2 Plot of System Cost for Three Interdependent Projects (Case 2)

4.2 Scheduling

Under the assumption that the benefits associated with a given combination of projects in some period vary only with the output of the system in that period, the start dates of the projects do not affect the system costs. Thus the SC curves for a project combination depend only on the presence, rather than start times, of particular projects in that combination. The implications in the context of waterways are that the capital cost of construction, operating and maintenance costs, and benefits from reduced delays are not affected by the age of the locks at any given time (i.e. by project start dates) but only by the volume of traffic using the locks. This assumption is very reasonable for the capital costs, but somewhat simplifies the operating and maintenance costs. The assumption is also reasonable for delay benefits although it neglects the effect of long term economic changes induced by the presence and performance of waterway investments.

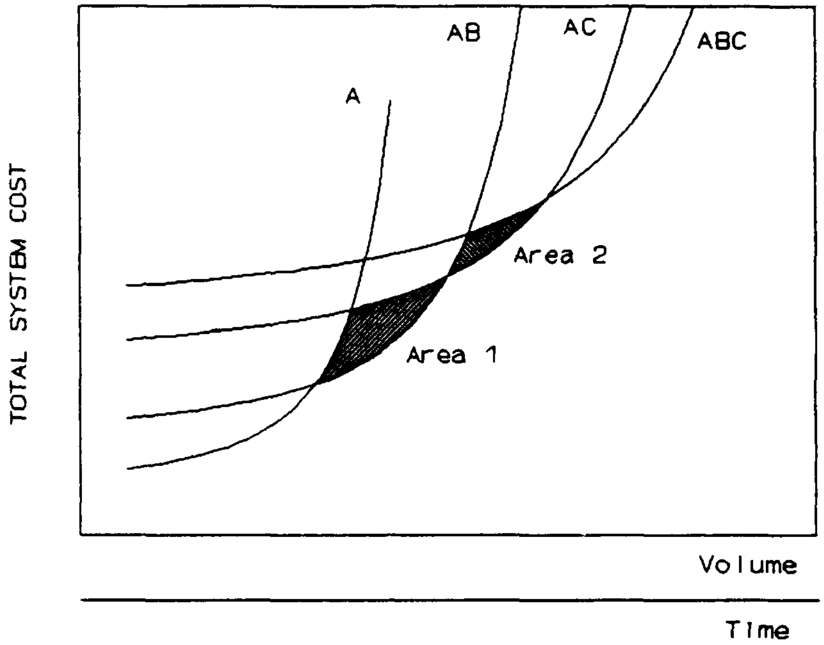

Figure 3 Plot of System Cost for Three Interdependent Projects (Case 3)

4.3 Incorporating a Budget Constraint

The representation of project combinations proposed thus far has not incorporated the effects of a budget constraint. In structuring the budget constraint, it will be assumed that funds not spent in a given period will be available in subsequent periods. This assumption is expressed by Equation 2 and is true for the Inland Waterway Trust Fund. For example, if $5 million is available and nothing is implemented in Period 1, then the $5 million is added to the budget limit for Period 2. Under this assumption, budget limitations have the effect of delaying the earliest feasible start date of a given project combination, just as they limit the earliest start of an individual project. Consider the small example of two projects A and B. In constructing the Curves A, B, and AB, the infeasible portion must not be included. Figure 4 illustrates that Combination A is not financially feasible until time T_1 corresponding to output Q_1. Combination AB is not feasible until time T_2. The three possible expansion paths are then as follows:

1. start A at time T_1 and B when Curves A and AB intersect
2. start B immediately and A when Curves B and AB intersect.

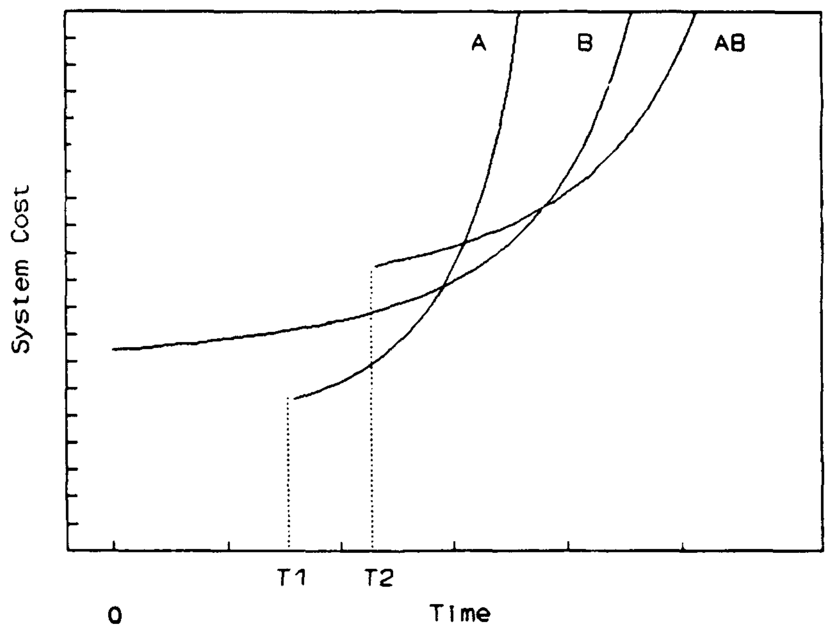

Figure 4 Incorporating a Budget Constraint

5. Sequencing and Scheduling Algorithm

The algorithm for selecting, sequencing and scheduling a subset of projects from a set of candidate projects searches for the minimum cost expansion path on a two dimensional plot of system cost functions for various combinations. The algorithm begins with an initial ranking of all projects based upon an evaluation where interdependencies among projects are neglected. This initial ranking is then modified iteratively to account for the interdependencies among the projects. The ranking of projects represents a sequence of search steps. Beginning with the null alternative, the next ranked project in the sequence is added to the provisional expansion program at each search step. At each implementation step, consideration is given to swapping two (or more) of the projects in the sequence. This is done by comparing the total system cost savings in a situation represented in Figure 5.

Total system cost curves represent the sum of delay and capital costs at all lock sites. It is computed for any set of independent as well as interdependent projects. The costs are then totaled across all interdependent projects and added to the costs of the independent projects to obtain the total system cost.

At each search step, four system cost curves are plotted. At the first search step (shown in Figure 5), the four combinations (project subsets) are C_1) no projects (null), C_2) null plus first ranking project, C_3) null plus second ranking project, and C_4) first and second ranking project, Figure 5. These four curves define two possible paths, 1) null-first-(first & second) or 2) null-

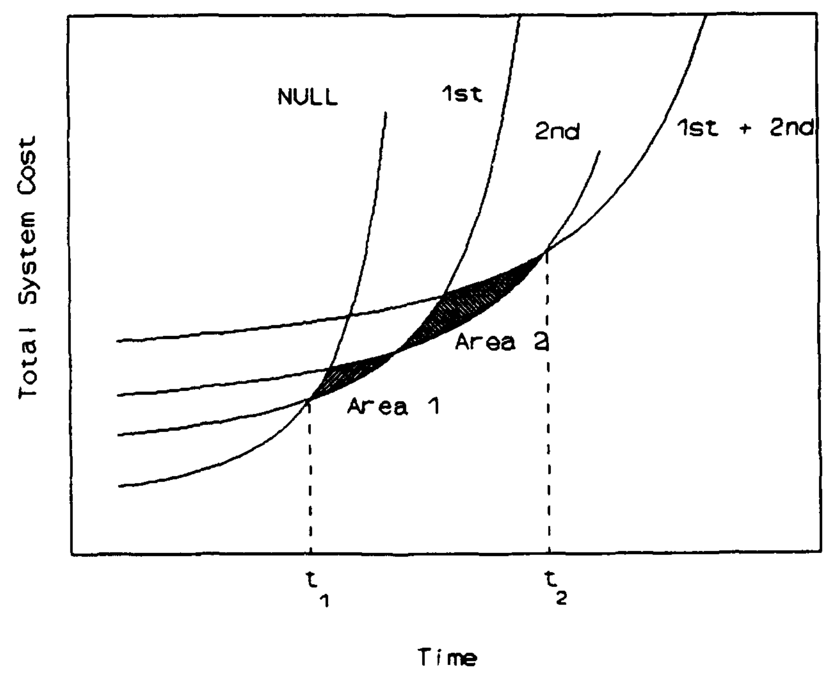

Figure 5 Plot for First Implementation Step

second-(first & second). The first path represents no change from the independent ranking, while the second path represents a swap between the first and second ranked projects. The selection between the two paths is based on the relative cumulative costs.

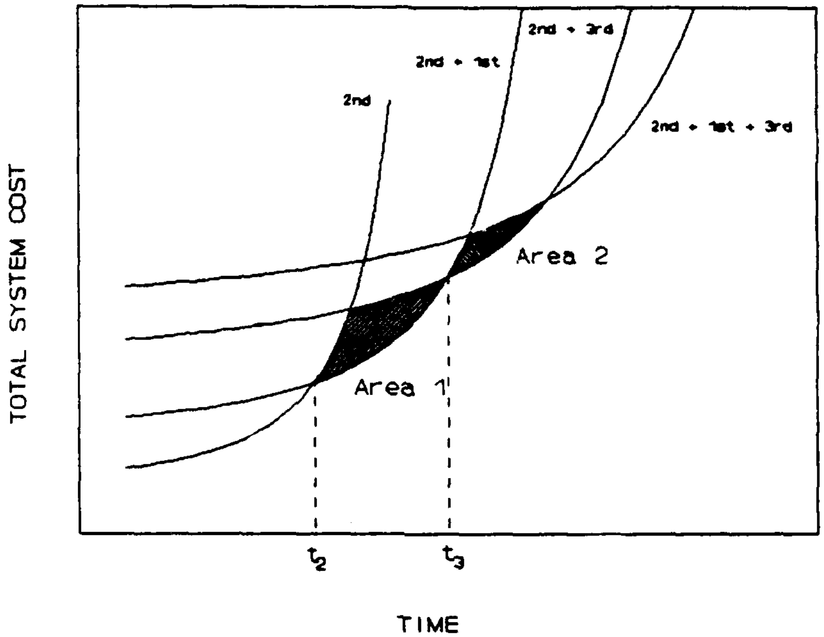

Figure 6 Plot for Second Implementation Step

If a swap to a higher ranking is found to be desirable for a project, then the additional swaps to successively higher rankings are considered for that project, until no further swap is desirable. Following all possible swaps, a new ranking is established. Subsequent implementation steps plot C1) corresponding to the current implementation set, C2) corresponding to the addition of the current first ranked among remaining projects, C3) corresponding to the addition of the current second ranked among remaining projects, and C_4) corresponding to the addition of both the current first and second ranked projects. For example, if a swap occurred during the first search step, then Figure 6 shows the combinations that would be plotted during the second search step. The start times for the projects are determined directly from the plots at each implementation step. The procedure is described in more detail in the subsections that follow.

5.1 Initial Ranking of Projects

The initial sequence is based on a relative evaluation of the projects assuming that they are independent. The evaluation may be made on the basis of the BCR. The benefits of improvement are a reduction in delay associated with the increased capacity, while the costs are the capital cost of construction. The BCR of a project i, that is independent from all others, may be written as

$$BCR_i = \sum_{t=0}^{h} ((I_0(t) - I_1(t)) \lambda(t)_i O_W(t)/K_i \qquad \text{Eq. 15}$$

where $I(t) \leq T$.

The function $I_0(t)$ is the delay before improvement, while $I_1(t)$ is the delay function after improvement. $I(t)$ may be determined analytically or experimentally. It should be noted that the project index on $\lambda(t)_i$ implies that volume may vary with time. Also, the opportunity cost of delay may be project-specific and be a general function of time. The constant T refers to a congestion tolerance in hours per tow. Tows will divert to other modes or waterways when delays reach $T + \epsilon$. After computing BCR_i for all projects, the projects are ranked according to decreasing BCR. This is the initial ranking (sequence) of the projects. It should be noted, that any project that has a BCR less than 1.0 should be eliminated from the expansion program at this stage of the analysis. This is because the presence of interdependence will only tend to lower a project's BCR (Martinelli and Sconfeld 1992).

5.2 A Routine for Modifying the Initial Sequence

The routine described in this section for modifying the initial sequence may be either coded algorithmically or performed interactively through a high-level programming environment. The routine begins with an initial ranking/sequence of projects, described by the n dimensional vector $\mathbf{R}_i$, e.g. $\mathbf{R}_i(2)$=A indicates that project A is second in the initial sequence. Project combinations are represented by a vector C of indices referring to a subset of projects in the vector $\mathbf{R}_i$. For example, if $\mathbf{R}_i$=(A,B,C,D), then the combination ABD would be expressed by the vector C=(1,2,4). The combination corresponding to the null alternative is denoted by C=(0). Finally a scheduling vector **T** is defined as the vector of start times corresponding to the projects in the sequence vector **R**. For example, **T**(3)=20 indicates that the third project in the sequence begins in time Period 20. The final sequence and schedule is represented by the vectors $\mathbf{R}_n$ and $\mathbf{T}_n$, respectively.

Following an initial ranking, the first project in the initial sequence is then tested for a possible swap with the second. A possible swap of the first project with the third project may also be considered. However, the computational requirements of the routine increase with the number of possible swaps considered at each iteration. In the interest of clarity, the description in this section is for consideration of one swap at each step.

The following are the steps for sequencing and scheduling a set of interdependent projects in which two or more are interdependent. The number of iterations in the routine is equal to one less than the total number of projects and is indexed by i.

<u>Step 1</u>

Compute the benefit cost ration (BCR) according to Equation 15 for each lock, assuming locks are independent. Rank in descending order. Let the initial sequence vector, $\mathbf{R}_0$, equal this ranking. $\mathbf{T}_0$ is initialized to all zeros. Let i=0. Go to Step 2.

<u>Step 2</u>

If i=0, then "plot" the interdependent system cost (TSC) curves for combinations C_1=C(0), C_2=C(1), C_3=C(2), C_4=C(1,2) versus time. If i ≥ 1, then plot the curves for C_1=C(1,..,i), C_2=C(1,..,i,i+1), C_3=C(1,..,i,i+2), C_4=C(1,..,i,i+1,i+2). The curves begin at a time corresponding to the availability of revenues equal to the sum of capital costs of the projects contained in a given combination.

$$B(t) = \sum_{j \in C} K_j \,/crf \qquad \text{Eq. 16}$$

The purpose of the plotting in this step is to locate the values of t for which five intersections may take place. These intersections are, t_1: C_1 and C_2, t_2: C_1 and C_3, t_3: C_2 and C_3, t_4: C_2 and C_4, and t_5: C_3 and C_4. The intersections help define areas for combination comparison such as Areas 1 and 2 in Figure 3 While these intersections may be determined visually if plotted, they may also be determined numerically without plotting. Numerical procedures for efficiently locating the intersections of convex functions currently exist.

The budget constraint is initially considered in this step of the routine. Revenues for major rehabilitations of lock facilities come primarily from the Inland Waterway Trust Fund and the federal matching share. Unspent funds accumulate according to a specified account interest rate. Because the start times for projects are on a continuous rather than discrete scale, the effect of the budget constraint is to delay the earliest possible start time for each combination. Therefore the times corresponding to the five intersection points, t_1, t_2,...,t_5, will be replaced with t'_1, t'_2,...,t'_5. Because a budget limitation may delay the start of a combination, combination curves that would otherwise intersect, will have no intersection. Therefore, times t'_1, t'_2,...,t'_5 do not correspond to intersections but rather to the adjusted earliest start time for a combination.

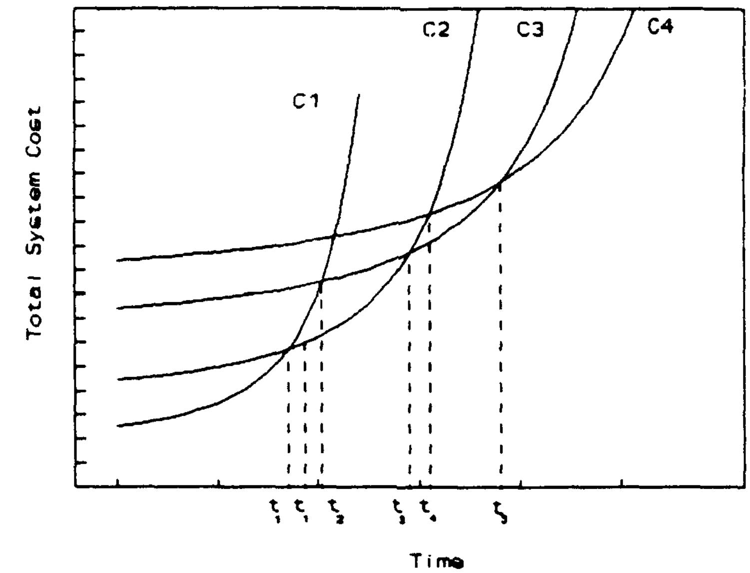

Figure 7 Quantities Obtained in Step Two

Figure 7 illustrates an example of the quantities obtained in this step. First, combination C_1 represents the current subset whose sequence has already been determined in previous steps. Combination C_2 represents the resulting combination if the current sequence is maintained, while C_3 represents the resulting combination if a swapping were performed at this step. Combination C_4 represents the implementation of the following project in the current sequence. In this example, the budget constraint has delayed the earliest start time for combination C_2 to t'_1. Therefore Project R(i+2) cannot start at t_1.
Go to Step 3.

<u>Step 3</u>

Evaluate to determine if Project R(i+1) should be swapped with R(i+2) in the sequence. This corresponds to a comparison between C_2 and C_3. The comparison is based on relevant areas defined by the system cost curves and the intersection points found in Step 2. If a swap is made, then a swap between project R(i+2) and R(i) is considered by redefining C_2 from C(1...i,i+1) to be C(1...i). If a second swap is made, then an additional swap between R(i+2) and R(i) is considered by setting C_2 to $C(1...i-1)$. Swaps are iteratively considered until a comparison is made where no swap is necessary.

In each case, unless one curve lies completely above the other, there will be an area that will favor $R(i+1)$ and an area that will favor $R(i+2)$. The evaluation is divided into four cases. The following tests may be used in making the correct comparisons for evaluation:

a. If the capital costs for combination C_2 are greater than for C_3 <u>and</u> the minimum capacity in C_2 is less or equal to that of C_3, then $R_i(i+1)$ <u>is</u> swapped with $R_i(i+2)$.

b. If the capital costs for combination C_2 are less than for C_3 <u>and</u> the minimum capacity in C_2 is greater or equal to that of C_3, then $R_i(i+1)$ is <u>not</u> swapped with $R_i(i+2)$.

However, if a budget limitation delays the earliest start for Combination C_2 to t'_1 then, if $t'_1 > t_2$, the following condition must be tested for a possible swap between $R_i(i+1)$ and $R_i(i+2)$:

$$\int_{t0}^{t'1} TSC(t)_{c1}\, dt + \int_{t'1}^{t5} TSC(t)_{c2}\, dt > \int_{t0}^{t2} TSC(t)_{c1}\, dt$$

$$+ \int_{t2}^{t4} TSC(t)_{c3}\, dt + \int_{t4}^{t5} TSC(t)_{c4}\, dt \qquad \text{Eq. 17}$$

In addition, if the budget delays the start of Combination C_4 to t'_5 then the third term on the right hand side (RHS) of Equation 17 is omitted and t_5 replaced with t'_5 in the integration limits.

Figure 8 is an illustration of the effects associated with a change in the earliest start time of Combination 2 due to the budget constraint. Without the budget constraint, path C_1-C_2-C_4 starts at 0 with C_1, switches to C_2 at t_1 and switches to C_4 at t_5. However, since funds are not available to begin C_2 at t_1, the system must continue under C_1 until t'_1. Because $t'_1 > t_2$, path C_1-C_2-C_4 is no longer entirely below path C_1-C_3-C_4. Therefore, the test of Equation 17 is necessary to determine which path yields the lower cumulative system costs.

c. If the capital costs for combination C_2 are less than that of C_3 <u>and</u> the minimum capacity in C_2 is less than or equal to that of C_3, then if the condition of Equation 18 holds (Area 1 < Area 2), $R_i(i+1)$ <u>is</u> swapped with $R_i(i+2)$.

$$\int_{t1}^{t2} (TSC(t)_{c1} - TSC(t)_{c2})dt + \int_{t2}^{t3} (TSC(t)_{c3} - TSC(t)_{c2})dt \quad <$$

$$\int_{t3}^{t4} (TSC(t)_{c2} - TSC(t)_{c3})dt + \int_{t4}^{t3} (TSC(t)_{c4} - TSC(t)_{c3})dt \qquad \text{Eq. 18}$$

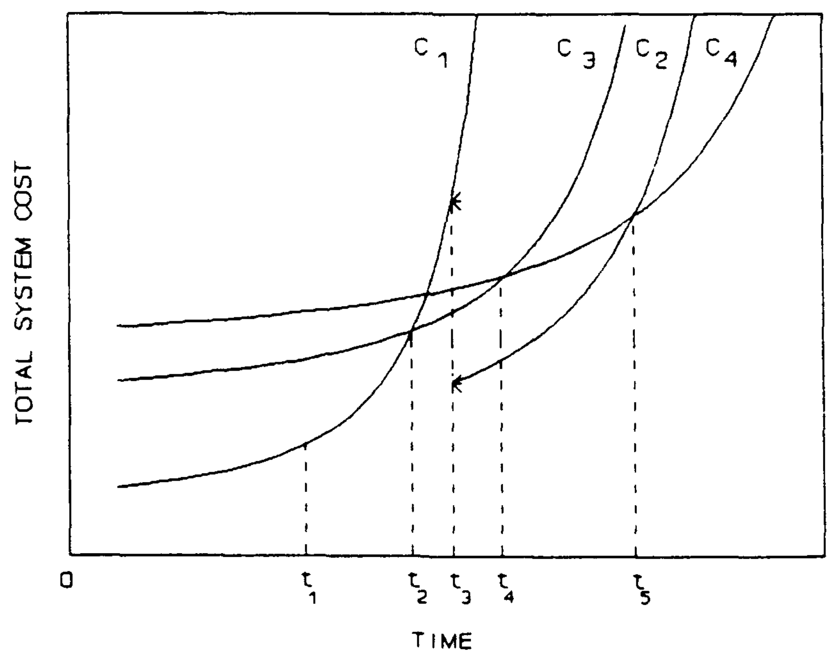

Figure 8 Illustration of Budget Effects (Case 1)

If the budget constraint alters t_1 and/or t_2, then the LHS of Eq. 18 (Area 1) is decreased by an amount given in Eq. 19 and increased by an amount given in Eq. 20.

$$\int_{t1}^{t'1} (TSC(t)_{c1} - TSC(t)_{c2})dt \qquad \text{Eq. 19}$$

$$\int_{t2}^{t'2} (TSC(t)_{c1} - TSC(t)_{c3})dt \qquad \text{Eq. 20}$$

Figure 9 is an illustration of the effects of the budget constraint if the earliest start time of C_3 is delayed from t_2 to t'_2. Here, path C_1-C_3-C_4 follows C_1 for an additional period of time (t'_2-t_2) yielding a higher cumulative cost than the same path without the budget constraint. This causes an increase in the size of Area 1.

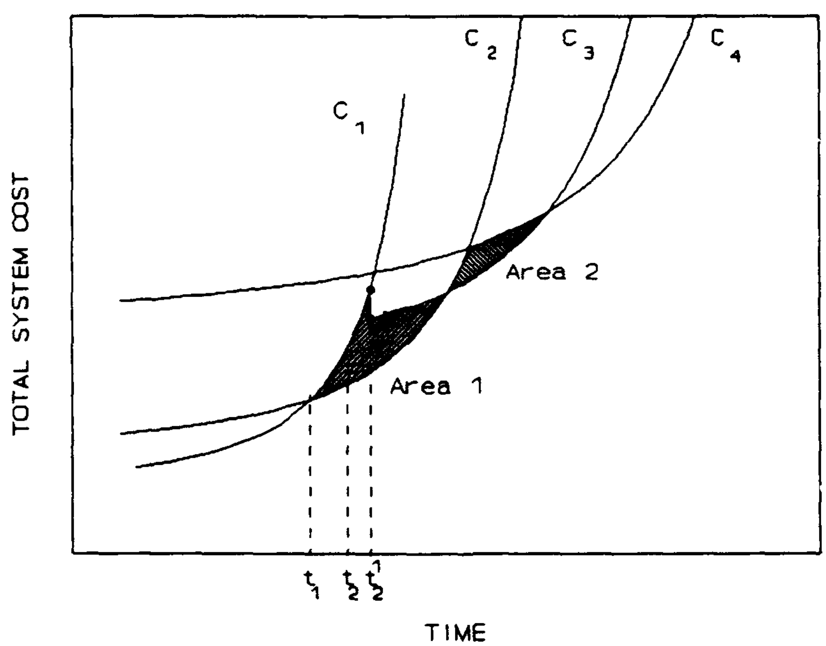

Figure 9 Illustration of Budget Effects (Case 2)

d. If the capital costs for combination C_2 are greater than that of C_3 <u>and</u> the minimum capacity in C_2 is greater or equal to that of C_3, then if the condition of Eq. 19 holds, $R_i(i+1)$ <u>is</u> swapped with $R_i(i+2)$.

$$\int_{t2}^{t1} (TSC(t)_{c1} - TSC(t)_{c3})dt + \int_{t1}^{t3} (TSC(t)_{c2} - TSC(t)_{c3})dt <$$

$$\int_{t3}^{t5} (TSC(t)_{c3} - TSC(t)_{c2})dt + \int_{t5}^{t4} (TSC(t)_{c4} - TSC(t)_{c2})dt \qquad \text{Eq. 21}$$

If the budget constraint alters t_1 and/or t_2, then the LHS of Equation 21 (Area 1) is decreased by an amount given in Equation 22 and increased by an amount given in Equation 23.

$$\int_{t1}^{t'1} (TSC(t)_{c1} - TSC(t)_{c3})dt \qquad \text{Eq. 22}$$

$$\int_{t2}^{t'2} (TSC(t)_{c1} - TSC(t)_{c2})dt \qquad \text{Eq. 23}$$

In general, TSC(t) is not an integrable function. However, as an approximation, the integrals may be replaced with summations and the limits rounded to the nearest integer value of t. Even better approximations are available through the rule of trapezoids or other numerical techniques. If the combination curves are plotted graphically, it should be possible to compare the sizes of Area 1 and Area 2 visually.

Go to Step 4.

<u>Step 4</u>

The sequence vector, R_i is updated for iteration i+1. If Projects R(i+1) and R(i+2) were not swapped in Step 3, then $R_{i+1} = R_i$. If the projects were swapped then let

$$R_{i+1}(i+1) = R_i(i+2) \qquad \text{Eq. 24}$$

and

$$R_{i+1}(i+2) = R_i(i+1). \qquad \text{Eq. 25}$$

Go to Step 5.

Step 5

The scheduled start time for the next project in the sequence is obtained directly from one of the intersection points found in Step 2. The following rules apply when assigning the start time for the next project in the sequence:

a. if Project R(i+1) was not swapped with R(i+2), then $T(i+1) = t_1$

b. if Project R(i+1) was swapped with R(i+2), then $T(i+1) = t_2$.

If i=n-1, then stop. Current sequence and schedule are final, otherwise increment i and return to Step 2.

5.3 Summary of Routine

The flow chart in Figure 10 provides a summary of the sequencing and scheduling routine described in this section. First, the user specifies the relevant data for the problem. These include 1) lock characteristics such as capacity, volume, distance from the previous lock, and growth rate, 2) project data such as construction costs and capacity improvements, and 3) other information such as planning horizon, interest rates, and congestion tolerance factor. This information is then employed to establish an initial ranking based upon an independent evaluation.

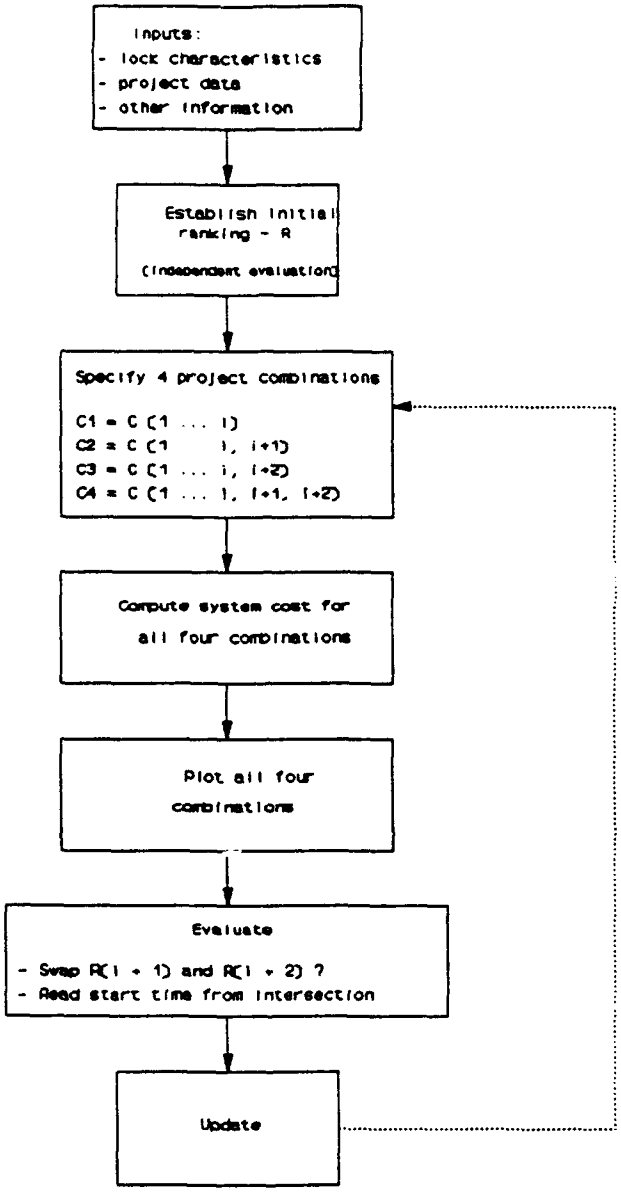

Figure 10 Flow Chart of Sequencing and Scheduling Routine

At the first iteration, the null alternative and three combinations involving the first two projects in the sequence are specified. Step 2 of the routine describes this specification step for subsequent iterations. Next, the total (delay and construction) system cost is computed for each combination for all time periods. The cost of each of the four combinations is plotted versus time on a single graph. Using the plot, the user determines whether the path C_1-C_2-C_4 or C_1-C_3-C_4 is less costly. If path C_1-C_3-C_4 has the lower cumulative cost, while considering the budget constraint, then projects R(i+1) and R(i+2) are swapped in the sequence, and the sequence is updated accordingly. An additional swap between R(i+2) and R(i) is then considered, followed by additional comparisons if swaps are continually made. Also, if a swap occurred, then the start time for project R(i+2) is read from the intersection of C_1 and C_3, else the start time for project R(i+1) is read from the intersection of C_1 and C_2. If the end of the project list is not reached, the user specifies the next set of four combinations, and repeats the plotting and evaluation steps.

6. Validation of Sequencing Algorithm

The algorithm presented is not theoretically guaranteed to yield the optimal sequence of projects. Therefore it is necessary to conduct an empirical validation of the algorithm. For validation, two experiments, one with four locks and one with six locks were conducted. In each experiment parameters of lock systems and proposed projects were randomly generated. The algorithm was then applied to each case to obtain a project sequence. Associated with the sequence is a cumulative system cost over the 40 year planning horizon. This sequence is compared to the optimal sequence, i.e. the sequence with the minimum cumulative system cost. The optimal sequence is determined through exhaustive enumeration of possible sequences. There are n! possible sequences, 24 and 750 for four and six lock cases respectively. There were 30 cases tested for four lock systems and 20 cases tested for six lock systems. The evaluation function used in the experiment is a one-directional metamodel (i.e. a model estimated statistically from simulation results) obtained from a related study (Dai and Schonfeld 1991).

Cases of locks and projects were randomly generated according to a uniform distribution. Ranges of variables were set in such a way as to guarantee a significant amount of interdependence. For example, the distance between locks for four lock systems is fixed at 10 miles and is between 5 and 20 miles for six lock systems. These low distances are not very common for the actual inland waterway system, but they yield a higher level of interdependence for a more challenging test of the sequencing algorithm. Table 1 shows the range of the randomly generated problem parameters.

The results of the experiments are tabulated in Appendix III. For each of the two experiments, there is a table providing the inputs for each case and a table providing the outputs for each case. The tables of inputs show the capacities before and after improvement, the capital cost of improvement for each lock, the initial volume level, growth rate, opportunity cost of delay, and distance between locks. Finally, the resulting BCR for the independent evaluation is given for each project. The output tables contain the sequences based on independent evaluation, the sequences based on the algorithm, and the optimal sequence obtained from exhaustive enumeration. Also given are the cumulative system cost of the algorithm sequence and the optimal sequence. From these costs an error may be computed for each case.

Table 1 Range of Problem Parameters for Sequencing Validation

Lower Limit	Parameter	Upper Limit
5	Initial Volume	35
.3	Initial Utilization	7
1%	Annual Growth Rate	5%
1.5	μ' / μ_0	2
$100	Opportunity Cost	$500
5	Distance	20

While the number of cases analyzed in each experiment is not extraordinarily high, the results suggest that the algorithm is likely to be effective in yielding an efficient project sequence. As the results in Appendix III indicate, the four lock experiment had two cases in which the algorithm did not successfully yield the optimal sequence. This is a success rate of 93.3% for the 30 cases. The two suboptimal cases had cumulative costs that were 0.8% and 2.3% higher than optimal. It appears that even when the algorithm does not yield an optimal solution, it does yield a reasonably good sub-optimal solution. Similar results were obtained for the six lock experiment. Specifically, the algorithm failed to yield the optimal sequence in one of the 20 six lock cases. This is a success rate of 95%. The error associated for the one suboptimal case was 4.1%.

By examining the cases were the algorithm is in error, ideas for improvements in the algorithm may be obtained. For example, Case 12 of the six lock experiment yielded an incorrect sequence resulting in a cost error of 4.1%. A close examination of this case reveals where in the algorithm the error was made. The algorithm yielded a sequence of 1,4,3,5,6,2 while the optimal sequence is 1,4,3,2,5,6. Applying the algorithm by hand on this case revealed that a swap between Projects 2 and 6 was considered, but the swapping criterion was not satisfied. If we go against the algorithm and consider a swap of Projects 2 and 5, we find that the swapping criterion is satisfied. However, the algorithm on its own does not attempt a swap between 2 and 5 because no swap was performed between 2 and 6.

There may be some adjustments in the algorithm which might eliminate such errors with little cost in computation time. For example, in a given iteration, swaps beyond the first unsuccessful swap may be considered. In other words, if a lower ranked project is not swapped with its predecessor, a swap between it and the next highest project may be considered. In order to control the amount of additional computations associated with this change, some maximum percentage cost difference may be specified for the initial comparison as a condition for considering the next highest swap. In applying this change to Case 12, a swap between Projects 2 and 5 would be considered if the cost difference in considering the swap between Projects 2 and 6 is within a specified amount.

7. Conclusions

A mathematical programming model was introduced as a conventional formulation for capital budgeting of interdependent projects. This formulation was shown to be inadequate for the prioritization and scheduling of inland waterway improvement projects. An alternative formulation that addresses the shortcomings of the conventional has been developed and shown to be effective for the application to inland waterway lock improvements. The method is conceptually and computationally straightforward, and has been show to yield promising results based on an experiment involving systems of four and six locks.

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Appendix II Notation

b	deviation in present value of benefits
B	Budget
BCR	Benefit Cost Ratio
c	deviation in present value of costs
C	project capital costs (conventional formulation)
C	combination of projects to be implemented
d	deviation from net present value
D	unspent budget
g()	generalized cost
I()	delay function assuming independence
K	project capital costs (new formulation)
O	opportunity cost of delay
NPV	Net Present Value
R	project ranking
SC	System Costs (capital and delay)
T	planning horizon; congestion tolerance
TNPV	Total Net Present Value
y	0,1 decision variable for project implementation
$\lambda, \lambda()$	output level (arrival rate at lock)

Appendix III Results of Sequencing Validation

Table 1 Outputs for Four Lock Experiment

Case	SEQ INDEP.	SEQ ALG.	SEQ OPT	Cost ALG.	Cost OPT.	Error
1	3,2,4,1	4,2,3,1	4,2,3,1	2.80×10^9	2.80×10^9	0
2	3,2,4,1	3,2,4,1	3,2,4,1	4.27×10^9	4.27×10^9	0
3	1,3,2,4	3,1,4,2	3,1,2,4	6.16×10^9	6.13×10^9	0
4	1,4,2,3	1,4,2,3	1,4,2,3	2.07×10^9	2.07×10^9	0
5	1,2,3,4	1,2,3,4	2,1,3,4	2.56×10^9	2.56×10^9	0.8%
6	3,2,4,1	3,2,4,1	3,2,4,1	1.55×10^9	1.55×10^9	0
7	3,1,4,2	3,4,1,2	3,4,1,2	2.54×10^9	2.54×10^9	0
8	4,3,2,1	4,2,3,1	4,2,3,1	2.91×10^9	2.91×10^9	0
9	4,2,3,1	4,3,2,1	4,3,2,1	7.49×10^9	7.49×10^9	0
10	4,3,2,1	3,2,4,1	3,2,4,1	8.28×10^9	8.28×10^9	0
11	2,4,1,3	2,1,4,3	2,1,4,3	2.83×10^9	2.83×10^9	0
12	3,1,4,2	3,1,4,2	3,1,4,2	2.43×10^9	2.43×10^9	0
13	2,4,3,1	4,2,3,1	4,2,3,1	4.59×10^9	4.59×10^9	0
14	3,2,1,4	3,2,1,4	3,2,1,4	2.70×10^9	2.70×10^9	0
15	3,2,1,4	3,2,1,4	3,2,1,4	2.96×10^9	2.96×10^9	0

16	2,4,3,1	4,2,3,1	4,3,2,1	3.01×10^{9}	2.94×10^{9}	2.3%
17	1,3,2,4	1,3,4,2	1,3,4,2	2.37×10^{9}	2.37×10^{9}	0
18	3,1,4,1	1,4,2,3	1,4,2,3	6.99×10^{9}	6.99×10^{9}	0
19	4,2,1,3	4,2,1,3	4,2,1,3	1.86×10^{9}	1.86×10^{9}	0
20	2,1,3,4	2,1,4,3	2,1,4,3	1.00×10^{10}	1.00×10^{10}	0
21	3,1,4,2	3,4,1,2	3,4,1,2	3.38×10^{9}	3.38×10^{9}	0
22	3,1,2,4	1,3,4,2	1,3,4,2	2.83×10^{10}	2.83×10^{10}	0
23	1,3,4,2	1,3,4,2	1,2,4,2	1.94×10^{9}	1.94×10^{9}	0
24	2,4,1,3	2,4,1,3	2,4,1,3	3.9×10^{9}	3.9×10^{9}	0
25	2,4,1,3	4,2,1,3	4,2,1,3	1.73×10^{9}	1.73×10^{9}	0
26	1,2,3,4	2,4,1,3	2,4,1,3	4.99×10^{9}	4.99×10^{9}	0
27	4,2,3,1	4,2,3,1	4,2,3,1	3.62×10^{9}	3.62×10^{9}	0
28	1,2,3,4	1,2,3,4	1,2,3,4	2.05×10^{9}	2.05×10^{9}	0
29	1,2,3,4	2,1,3,4	2,1,3,4	7.33×10^{9}	7.33×10^{9}	0
30	2,1,3,4	1,3,2,4	1,3,2,4	5.22×10^{9}	5.22×10^{9}	0

Table 2 Outputs for Six Lock Experiment

CASE	INDEP SEQ	ALG SEQ	OPT SEQ	ALG COST	OPT COST	ERROR
1	241635	421356	421356	$5.17x10^9$	$5.17x10^9$	0
2	364152	345162	345162	$3.73x10^9$	$3.73x10^9$	0
3	432165	432165	423165	$2.08x10^8$	$2.08x10^8$	0
4	214365	132465	132465	$5.87x10^9$	$5.88x10^9$	0
5	135462	153426	153426	$5.31x10^9$	$5.31x10^9$	0
6	453612	421356	345126	$1.24x10^{10}$	$1.24x10^{10}$	0
7	452631	425136	425136	$6.22x10^9$	$6.22x10^9$	0
8	415236	154236	154236	$6.18x10^9$	$6.18x10^9$	0
9	231456	321546	321546	$6.31x10^9$	$6.31x10^9$	0
10	142563	241536	241536	$7.95x10^9$	$7.95x10^9$	0
11	526134	521346	521346	$6.71x10^9$	$6.71x10^9$	0
12	134652	143562	143256	$7.38x10^9$	$7.51x10^9$	4.1%
13	142356	514236	514236	$8.94x10^9$	$8.94x10^9$	0
14	134562	153642	153642	$5.00x10^9$	$5.00x10^9$	0
15	526431	524631	524631	$3.22x10^{10}$	$3.22x10^{10}$	0
16	451326	154236	154236	$5.23x10^9$	$5.23x10^9$	0
17	236145	321465	321465	$1.12x10^{10}$	$1.12x10^{10}$	0
18	423165	234153	234156	$8.60x10^9$	$8.60x10^9$	0
19	153642	351462	351462	$7.04x10^9$	$7.04x10^9$	0
20	132465	123465	123465	$6.47x10^9$	$6.47x10^9$	0

APPROXIMATING DELAYS AT INTERDEPENDENT LOCKS

by

David Martinelli and Paul Schonfeld

Approximating Delays At Interdependent Locks

David Martinelli, AM ASCE[1] and Paul Schonfeld, AM ASCE[2]

Abstract
As with much of the nation's infrastructure, the inland waterway system is in critical need of expansion and repair. Many of the inland waterway lock and dam facilities have become major constraints to navigation due to increased traffic and facility deterioration, leading to costly delays. Because funds for lock and dam improvements are severely limited, comprehensive analysis methods are necessary to ensure efficient allocation of resources among the many proposed improvement projects. Unfortunately, locks and dams are often treated as independent facilities with regards to operations when in fact, there are likely to be significant interdependencies among many of them. There is a need for an analysis technique that accounts for interdependencies between locks when considering lock improvements. In this paper, a method is developed whereby the delays of a set of interdependent locks may be calculated. By incorporating interdependencies into benefit calculations of lock improvement projects, a more comprehensive assessment of improvement priorities can be established.

Keywords
inland waterways, interdependence, queues, simulation, metamodel

Summary
A method for computing delays for interdependent locks is developed through the formulation of evaluation functions based on the results of a simulation experiment. The method could help lead to more comprehensive project evaluation and prioritization of lock rehabilitation projects.

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1. Introduction

Inland waterways are an important part of the nation's transportation network. Approximately 16 percent of the intercity freight in the U.S. moves by waterway. Coal, petroleum products, and grains are the top three tonnage commodities, accounting for about 60 percent of the inland waterway commerce. The National Waterways Study (NWS) forecasts an increase in total U.S. waterborne traffic from 1,915 million tons in 1977 to a 2,890 million tons by 2003 (USACE 1987). Locks and dams are essential for creating stepped navigational pools with reliable depths for navigation. However, many of these facilities have become major constraints to inland navigation, due to increased traffic and facility deterioration, leading to costly delays.

Prediction of lock delays is important for, among other things, evaluating rehabilitation and reconstruction alternatives. Each lock, however, is part of a system of locks, and in general, its delays may be dependent on operations at one or more other locks. These interdependencies limit the value of available tools for predicting lock delays. Specifically, the queuing process at locks cannot be modeled as an M/M/1, or even M/G/1, process. Moreover, locks typically have more than one chamber and do not operate under a first-in-first-out queue discipline. Therefore, queuing theory does not provide any satisfactory analytic solution for predicting lock delays.

The objective of the research described in this paper has been to develop a technique for approximating average delays at inland waterway locks while considering their interdependencies. The ultimate use of such a technique is to evaluate the benefits associated with rehabilitating given combinations of locks for investment planning. While a microsimulation model for capturing such delays is available (Dai and Schonfeld 89), a significant amount of computer time is required for variance reduction, when obtaining reliable predictions of delays. Furthermore, for investment planning purposes, it is necessary to execute such a model for numerous combinations of reconstruction projects. Therefore there is a need to develop a functional simplification that will adequately substitute this simulation and perform satisfactorily as an evaluation tool for investment planning.

2 Defining Lock Improvement Interdependencies

If locks are independent, then however a lock operates, it does not affect the performance of any other locks upstream or downstream. Then the total delay of a system of locks is no different from the sum of the delays of each lock acting in isolation. On the other hand, if interdependencies between two or more locks in a system exist, then the total delay of the system will be different from the sum of the isolated delays. If the total system delay, S, is equal to the total isolated delays, I, then the locks may be considered independent. Therefore, a ratio of system delay to isolated delay, S/I, may serve as a *theoretical* measure of the total interdependence in the system. If S/I = 1, then there are no interdependence effects, and if S/I < 1 then interdependence effects exist. It may be noted that S/I cannot exceed 1. It should also be noted that we are concerned only with assessing the *total effects* of interdependence and not an itemization of all component interdependencies.

It is possible to establish a theoretical lower bound for S/I. Consider a one directional system of two identical locks A and B. Also, assume the spacing between vessels to be unchanged when traveling between Locks A and B and the lock service times are not random variables. Although there may be delays, W_A at Lock A, there will not be any delays at Lock B, due to a metering effect from Lock A. In other words, tows will arrive at Lock B at intervals that exactly correspond to service times from A. However, if each lock is considered in isolation, then the delays at B will equal the delays at A, yielding a total delay of $2W_A$ for the two lock system. Therefore the lower bound on S/I for a two lock system is 1/2. In general, for similar systems of n locks which represents a lower bound on S/I on S/I (or an upper bound on the amount of interdependence in the system). For the more realistic stochastic case, S/I will be larger than 1/n due to randomization opportunities for arrival patterns and service times at Lock B.

$$\frac{S}{I} = \frac{1}{n} \qquad \text{Eq. 1}$$

3. Possible Factors Affecting Lock Interdependence

There are numerous factors that might be related to lock interdependence. For example, a Pennsylvania State University study classified locks as dependent or independent according to the linehaul distance separating them (Carrol 1972). Queuing theory is helpful in identifying the most relevant factors. Using an M/G/1 queuing model, the average wait time, W, has been reported in (Whitt 84) to be

$$W = \frac{1}{\mu} + \frac{\rho^2 + \lambda^2 \sigma^2}{2\lambda(1-\rho)} \qquad \text{Eq. 2}$$

where μ and λ are the mean service and arrival rates respectively, ρ is lock utilization, and σ is the standard deviation of the service time distribution. In examining this queuing model, as well as work by others, several possible factors relating to delay interdependence may be identified. In a series of locks, if the arrivals at a particular lock are Poisson distributed, then it is likely that the effects from the previous locks have been diluted. In other words, interdependence is related to the *opportunity* (or lack thereof) for vessels to "randomize" between locks. Linehaul distance, speed (mean and variance), volume, passing opportunities, and network geometry are all likely to affect this opportunity. Similarly, factors may be identified from the service process such as utilization, relative utilizations, tow size distributions, queue discipline, size and number of chambers, and lock reliability.

4. Simulation Experiment

The stochastic nature of the lock delay problem under assumptions of generalized arrival and service distributions may limit the scope of an analytic model. It is indeed quite difficult to develop expressions for average delays under these conditions while capturing the effects of interdependencies. For this reason, a simulation model proves to be a viable approach to obtaining realistic relationships between certain factors and interdependence among locks. Simulation models are appropriate when a complete mathematical formulation of the problem does not exist or analytical methods require excessively restrictive assumptions.

The Transportation Studies Center at the University of Maryland has developed a microscopic waterway simulation model to analyze the relationships between tow trips, travel times, delays, and lock operations [Dai and Schonfeld 89]. The model traces the motion and records the characteristics of each tow (e.g. number of barges, commodity type, speed, origin, destination, direction, and arrival times), while allowing for variability in many of the lock queuing factors such as capacity, volume level, etc.

The model is event scanning, i.e. the status is updated by the occurrence of one of five events 1) trip generation, 2) tow entrance at locks, 3) tow arrival at destinations, 4) lock stall, and 5) end of inventory period. The model places no restrictions on the number of locks, chambers, cuts, waterway links, tows, O-D pairs, and time periods. One limitation of the model, however, is that it currently has only been validated for series network geometry. The validity of the model has been tested by comparing the model predictions with actual data along five Ohio River locks. Traffic volumes were predicted quite accurately by the model, with an average deviation of 1.53%. The waiting times at locks were predicted within a 10% error. The estimates of these quantities were made without any systematic bias. Therefore, the simulation model, which is based on real (obtained from the Lock Performance Monitoring System) data, can act as a surrogate in an experiment involving variables believed to be important in lock interdependence.

As is common with micro-simulation, the model developed by Dai & Schonfeld (1989) requires excessive computer time to evaluate numerous combinations of projects. In order to evaluate a combination of projects, it is necessary to compute the total average delays both at current capacity levels and improved capacity levels. Also, for variance reduction purposes, it is desirable to perform numerous runs (e.g. 30) for each observation. Because the number of possible combinations of projects may be large, a naive selection and sequencing technique would require the complete evaluation of numerous combinations. For most project proposal sets, using a micro-simulation model alone would be prohibitively expensive.

An alternative to direct application of micro-simulation is to employ the simulation model in an experiment to assess, in functional form, the degree to which certain explanatory factors contribute to lock interdependence. Specifically, an experiment was done to explore the extent to which delays at a particular lock are affected by changes in the characteristics of other locks. The data generated from the experiment were employed to estimate functions to be used as a substitute for the simulation model. A model estimated from simulated data is termed a metamodel (Law and Kelton 1982).

Because queuing delays are the source of interdependence, an experiment was designed involving a system of locks that have the total delay in the system as the *response* variable. *Factor* variables were then chosen on the basis of queuing theory and the opportunities for vessels to randomize between locks. The factor variables found relevant for the simulation experiment are:

Distance, D - is the linehaul distance between locks and is likely to be a strong indicator of the opportunity for the spacing between vessels to randomize. The larger this distance, the greater the vessel spacing randomization and the smaller the degree of interdependence.

Critical utilization, ρ_c - is the maximum volume to capacity ratio in the system and is a measure of the extent to which traffic is "metered" through a critical lock. Poisson distributed traffic (a condition for independence) may be distorted by the metering of traffic at a lock. Therefore a high critical utilization is likely to give rise to interdependence.

Relative utilization, U - is the ratio of the utilization of a given lock to that of the critical lock and measures the extent to which the delays at a given noncritical lock may be dominated by the delays at the critical lock.

Table 1 shows the values associated with each level of the factor variables used in the simulation experiment. The range of values included are derived from typical values observed in the inland navigation system. A combination of values of the factor variables represents a simulation case. For each case, the simulation model must provide the data necessary to compute S/I.

The experiment involved the simulation of various systems of two locks, labeled Lock 1 and Lock 2. Larger systems would not allow for as large a range of factors and levels. The basic system of locks simulated is shown in Figure 1. First, two locks with given levels of the factor variables were simulated as a *system*, where the interdependence is captured and included in the resulting average delay, (Fig. 1 top). Second, two independent locks with identical levels of the factor variables were each simulated as a one-lock system (Fig. 1 bottom). Specifically, the average total delay must be obtained for the locks acting as a system and for both locks acting independently. The ratio of these two totals is S/I.

To achieve the various levels of the factors for a two lock system, Lock 1 was considered critical, i.e. it always had the larger utilization. The capacity of Lock 1 was fixed at 60.6 tows per day and the system volume was adjusted to yield the desired utilization. For example, to achieve a critical utilization, ρ_c, of .89, the volume level used in simulation is 53.93 tows per day, since 53.93/60.6 = .89. The desired utilization of the second lock is obtained by the given level of U. For example, if U=.633 and ρ_c = .89, then the utilization of the second lock is .560. The utilization of the second lock is achieved by adjusting its volume. In this case, the volume of Lock 2 would be 33.94 because 33.94/60.6=.56. A summary of the utilizations and volumes of Lock 2 necessary to yield desired combinations of U and ρ_c is provided in Table 2.

An assumption of this simulation experiment for estimating evaluation functions is that the only geometric configuration considered is a series of locks. This assumption is reasonable given the near tree-structure of the inland waterway network. A possible expansion of this methodology would include simulation of the junction points. A conservative number of independent simulation runs per observation was found to be 30 in order to sufficiently reduce the simulation variance. A conservative number of tows required per simulation run to achieve this objective is on the order of 13,000. Typically in the simulation runs, the first 1000 tows were deleted to allow the system to stabilize and results were based on the next 12,000 tows in each run.

5. Experiment Results

Table 3 provides a sample of the data for utilization ratio of 1.00 and critical lock utilization of 0.89. Specifically, mean isolated and system waiting times are shown for each of the two locks for all three distance levels for directions 1 and 2. Also included are the total system and isolated delays for both locks over both directions. Finally, the corresponding value for the interdependence coefficient, S/I, is computed from the total system and isolated delays from traffic in both directions.

Some of the data are plotted in Figure 2 with S/I on the vertical axis and utilization of the critical lock on the horizontal axis. Each point in the plots represent the average of 30 runs. This and other plots were helpful in making first assessments of the functional form of the interdependence coefficient. It appeared from these plots, that the upper bound on S/I is slightly less than 1.0 and that it decreases at an increasing rate with the utilization of Lock 1. There was a noticeable relation between the level of interdependence and both the critical utilization and distance. The exploratory analysis suggested that a functional model for S/I should include relative utilization, critical utilization, and distance between locks as variables.

The simulation output suggests that as the distance between two locks *increases*, the amount of interdependence among those locks also *increases* (S/I decreases). This result is consistent with earlier studies, e.g. (Carrol 1972). In Section 3, it was mentioned that lock interdependence is inversely related to the opportunities for the intervals of vessels to randomize. Intuitively, the distance between locks tend to increase the randomization of traffic. On the other hand, distance may not be the predominate influence on interdependence as some studies suggest. For example, in Carrol (1972), distance was the only variable considered. Based on these results, it appears that the critical utilization has a more significant effect on interdependence than distance.

6. Functional Estimation for S/I

In this section, a mathematical function that is reasonably consistent with the compiled simulation data is derived. The experiment results suggest that the functional form of the interdependence coefficient is nonlinear in terms of the factor variables with an upper bound of 1.0. However, since a lower bound of 1/2 is based on a deterministic system, it may not be used in establishing a functional form. A functional form that yields 1.0

at a volume level of 0 and decreases faster than linearly with critical utilization would seem to closely fit the data as plotted. One tractable mathematical form expressing such a relation is given in Eq. 3:

$$\frac{S}{I} = 1 - \alpha\ \rho_c^{\beta}\ U^{\gamma}\ D^{\delta}. \qquad \text{Eq. 3}$$

Expanding the ρ_c and U terms for locks labeled 1 and 2 we obtain:

$$\frac{S}{I} = 1-\alpha\ (\max(\rho_1,\rho_2))^{\beta}\ [\min(\rho_1,\rho_2),\max(\rho_1,\rho_2)]^{\gamma}\ D_{12}^{\delta} \qquad \text{Eq. 4}$$

This relation may be interpreted to have an upper bound of 1.0 with the second term representing a quantity of interdependence to be subtracted. Conceptually, $\alpha\ \max(\rho_c)^{\beta}$ represents the maximum interdependence that may be possible, while U^{γ} and D^{δ} are multipliers that determine the portion of the possible interdependence that may be realized. The interdependence coefficient may then be used to compute the total system delay of a two lock system, S_{12}

$$S_{12} = \frac{S}{I}\ (1 + I_2). \qquad \text{Eq. 5}$$

In Eq. 5, I_1 and I_2 are the delays of the first and second locks acting in isolation, respectively. Later it will be shown how the values for I_1 and I_2 were estimated.

While the functional form for S/I is nonlinear, it is exponential and subject to logarithmic transformation.

$$\mathrm{Log}(1-\frac{S}{I}) = \log \alpha + \beta \log \rho_c + \gamma \log U + \delta \log D \qquad \text{Eq. 6}$$

The estimation results for this model are shown as numerical parameters in Eqn. 7. Converting the transformed variables to their original form yields the following estimated model for the interdependence coefficient of a two- lock system

$$\frac{S}{I} = 1 - 0.713\ (\rho_c)^{2.455}\ U^{0.944}\ D^{-0.506} \qquad \text{Eq. 7}$$

7. Queuing Nature of Independent Locks

In a one-directional series of n isolated locks, the first lock is independent of any previous lock. Therefore, the waiting time at the first lock may be described by either an M/G/1 queuing model, as shown by Burke's Theorem, or by a model estimated from simulation results for isolated locks. All other locks in the series will have a significantly more complex G/G/1 arrival process. If an M/G/1 process is assumed for the first lock, then the waiting time is expressed in Eq. 2. It may be more accurate to estimate a function for the delay at an independent lock from the simulation data. A comparative plot between the delays for an M/G/1 lock and the simulated data revealed that while the M/G/1 function is reasonable, an estimated function would be more accurate. One functional form that includes the same parameters as the theoretical expression for M/G/1 is the following:

$$W_I = \lambda^a \ (1-\rho)^b \ \sigma^c \qquad \text{Eq. 8}$$

Like the theoretical expression for M/G/1, this function is asymptotic to the capacity as shown by the $(1-\rho)$ term. The parameters for this function were estimated using the simulated data. Because this model is exponential, it also is subject to logarithmic transformation.

$$\log W_I = a \log \lambda + b \log \rho + c \log \sigma^2 \qquad \text{Eq. 9}$$

The estimated values for the parameters of the isolated delay model yields the following model

$$I = \frac{\sigma^{2.85}}{\lambda^{.413} \ (1-\rho)^{1.950}}. \qquad \text{Eq. 10}$$

8. Expanding from Two Locks to n Locks

The model for the interdependence coefficient, S/I, is sufficiently consistent with the results of the simulation experiment. Thus, for two lock systems, a satisfactory function has been developed to evaluate interdependent projects without the use of simulation (Eq. 5). However, it is likely that groups of locks in a series of three or more, are interdependent. In this section, a procedure for expanding the functional relationship for interdependent 2-lock systems to n-lock systems, such as shown in Figure 3, is described. Note that the assumption of a one-way system is necessary in developing the expansion.

One possible method is to sum the interdependence among the successive pairs in the system. To illustrate, Y_{12} is substituted in place of the second term in the expression for the interdependence coefficient and a subscript added to denote the number of locks in the system. The variable Y may be referred to as the interdependence variable since it represents the amount of interdependence among locks.

$$Y_{12} = \alpha \ (\rho_c)^{\beta} \ (U_{12})^{\gamma} \ (D_{12})^{\delta} \qquad \text{Eq. 11}$$

$$\left(\frac{S}{I}\right)_2 = 1 - Y_{12} \qquad \text{Eq. 12}$$

It follows that the coefficient for the three lock system would include a term for the interdependence between Locks 2 and 3.

$$\left(\frac{S}{I}\right)_3 = 1 - (Y_{12} + Y_{23}) \qquad \text{Eq. 13}$$

However, this way of summing interdependence has some shortcomings. First, the technique does not incorporate the variance in lock service times. Second, the interdependence among nonadjacent locks is not accounted for. To illustrate the technique used to overcome such shortcomings, the three lock system in Figure 4 is employed. The figure indicates that there is a variance, utilization, arrival process, and departure process associated with each lock. Distance D_{12} separates Locks 1 and 2 while D_{23} separates Locks 2 and 3.

Note that Lock 1 has an independent arrival process, while the arrival processes at the remaining locks are related to the departure processes from the previous locks. Thus, the delay for Lock 1 may be determined by using Eqn. 10. Enclosing Locks 1 and 2 in the figure establishes an *effective* lock, e2, which may act as a proxy for the delays of Locks 1 and 2 combined. Associated with the effective Lock e2 is also a variance, utilization, arrival process, and departure process are also associated with effective Lock e2. The arrival process of Lock e2 is the same as that of Lock 1, while its departure process is the same as that of Lock 2.

Note that Locks e2 and 3 constitute a two lock system, i.e. the 2-lock model for the interdependence coefficient estimated directly from the simulation experiment may be applied. If ρ_{e2} and σ_{e2} are found, then in terms of total delay, the three lock system will be successfully converted to an equivalent two lock system. Because the arrival process for Lock e2 is independent, ρ_{e2} is the utilization that yields the total system delay for Locks 1 and 2, S_{12}, but as an independent lock. Therefore Eq. 10 may be applied to compute the delay for Lock e2, S_{e2}.

$$S_{e2} = \lambda^{a} \ (1-\rho_{e2})^{b} \ (\sigma_{e2})^{c} \qquad \text{Eq. 14}$$

However, the delay for Lock e2, S_{e2}, is the same as the total system delay for Locks 1 and 2, S_{12}.

$$S_{e2} = S_{12} = 1 - \alpha \ \rho_c^{\beta} \ U^{\gamma} \ D^{\delta} = \lambda^{a} \ (1-\rho_{e2})^{b} \ (\sigma_{e2})^{c} \qquad \text{Eq. 15}$$

Next, an expression for the combined service time variance of Locks 1 and 2, $(\sigma_{e2})^2$ is obtained. If it is assumed that the service times of any group of n locks are independent, then the variances may be added linearly to yield the system variance, $(\sigma_{en})^2$.

$$\sigma_{en}^{2} = \sum_{i=1}^{n} \sigma_{i}^{2} \qquad \text{Eq. 16}$$

Therefore $(\sigma_{e2})^c$ may be replaced with $(\sigma_1^2 + \sigma_2^2)^{c/2}$ in Eqn. 15. The utilization of the effective lock, ρ_{e2} may now be solved directly from 15. Solving for ρ_{e2} we have

$$\rho_{e2} = 1 - (S_{12} \ \lambda^{-a} \ (\sigma_1^2 + \sigma_2^2)^{-c/2})^{1/b} \qquad \text{Eq. 17}$$

The interdependence coefficient may now be computed for a three lock system by adding the interdependence variable between Locks e2 and 3 to that of Locks 1 and 2, as illustrated in Figure 5. This is equivalent to replacing Y_{23} in Eqn. 10 with Y_{e3},

$$\left(\frac{S}{I}\right)_3 = 1 - (Y_{12} + Y_{e3}) \qquad \text{Eq. 18}$$

where

$$Y_{e3} = \alpha \max(\rho_{e2}, \rho_3)^{\beta} (U_{e3})^{\gamma} (D_{23})^{\delta} \qquad \text{Eq. 19}$$

and

$$U_{e3} = \frac{\min(\rho_{e2}, \rho_3)}{\max(\rho_{e2}, \rho_3)}. \qquad \text{Eq. 20}$$

The coupling technique may now be applied in succession to yield the S/I for an N-lock system, $(S/I)_N$. This is done by first computing Y_N starting with $Y_2 = Y_{12}$. The following is an algorithm for applying the technique in computing $(S/I)_N$.

Step 1

Begin with the two locks farthest upstream and compute the interdependence index between them.

$$Y_2 = \alpha \max(\rho_1, \rho_2)^{\beta} (U_{12})^{\gamma} (D_{12})^{\delta} \qquad \text{Eq. 21}$$

Using Y_2, compute the interdependence coefficient for the first 2 locks.

$$\left(\frac{S}{I}\right)_2 = 1 - Y_2 \qquad \text{Eq. 22}$$

Set n= 3.

Step 2

Compute the system delay for the first n-1 locks.

$$S_{n-1} = \left(\frac{S}{I}\right)_{n-1} \sum_{i=1}^{n-1} I_i \qquad \text{Eq. 23}$$

Step 3

Compute the combined standard deviation of service time for the first n-1 locks.

$$\sigma_{cn} = \sum_{i=1}^{n-1} (\sigma_i^2)^{1/2} \qquad \text{Eq. 24}$$

Step 4

Compute the effective utilization for the first n-1 locks.

$$\rho_{cn-1} = 1 - (S_{n-1}\ \lambda^{-a}\ (\sigma_{n-1})^{-c})^{1/b} \qquad \text{Eq. 25}$$

Step 5

Compute the interdependence index between Lock en-1 and Lock n.

$$Y_{cn} = \alpha \max(\rho_{cn-1}, \rho_n)^{\beta}\ (U_{cn})^{\gamma}\ (D_{n-1\,n})^{\delta} \qquad \text{Eq. 26}$$

Step 6

Compute the interdependence index for the n lock system.

$$Y_n = Y_{n-1} + Y_{cn} \qquad \text{Eq. 27}$$

Step 7

Compute the interdependence coefficient for the first n locks.

$$(S/I)_n = (S/I)_{n-1} - Y_n \qquad \text{Eq. 28}$$

If n=N, then stop, else increment n and go to step 2.

The above derivation for $(S/I)_n$ was performed *using* the statistically estimated formula for the delay at the first lock in the series, Eqn. 10. If an M/G/1 process is assumed for the first lock in the series, then ρ_{e2} is determined from Eqn. 29 where $W_{M/G/1}$ is replaced with S_{12}. The remaining steps in the technique are unchanged.

$$\rho_{e2} = .5(2 + 2\lambda S_{12})-[(2 + 2\lambda S_{12})^2 + 4(\lambda^2(\sigma_{e2})^c-2\lambda S)]^{1/2} \qquad \text{Eq. 29}$$

The results from the simulation experiment have now been expanded from incorporating systems with only two locks, to series with any number of locks. The expansion technique first assesses the interdependence of the first two locks estimated directly from the simulation results. Next, an additional factor of interdependence is added for the third lock. This additional factor is not based on the interdependence between Lock 3 and Lock 2 only, as Eqn. 13 would suggest, but rather is based on the interdependence between Lock 3 and the *system* composed of Locks 1 and 2. In a similar manner, terms for interdependence associated with additional locks are added one at a time as suggested by the iterative nature of Eqns. 21 through 28.

9. Validation of Lock Coupling

The coupling of locks can be used to obtain an S/I ratio that reflects the interdependence within a system of more than two locks. That ratio can then be used to calculate the total system delay among the locks. Therefore, the effectiveness of the coupling technique may be measured by the ability to yield values of system delay that are within acceptable deviations from simulated values.

To perform a test of the coupling technique, an experiment was conducted to provide simulated results for a three lock system. Comparisons of the total delay between the simulation model and the coupled metamodel provides some measure of effectiveness for coupling from a two lock system to a three lock system. Because the systems simulated were bidirectional, the experiment also provides an indirect validation of the one-directional assumption employed by the coupling technique.

The experiment involved three lock systems with the same utilization levels as the two lock simulation experiment, namely .890, .750, .660, and .320. A total of 40 three-lock combinations with these utilizations were simulated. The simulations were conducted at a constant volume level of 30 tows/day and the capacities of the locks were adjusted accordingly to yield the desired utilizations. The distance between locks was 20 miles in all cases. A range of standard deviations of service time, σ_s, were considered by holding the coefficient of variation, σ/μ, constant at 0.5. As with the two lock simulation experiment, there were 30 runs per each of the 40 observations and approximately 13,000 tows per run.

Appendix 2 summarizes the numerical results of the experiment. Tabulated are 1) the average delay from simulation observed at each lock, 2) the variance of the service time at each lock, 3) the computed delay of each lock in isolation, 4) the level of interdependence as measured by the S/I ratio for each three lock system, 5) the computed total simulated delay, 6) the computed total delay, and 7) the percent deviation from simulated results.

The average deviation from simulated results is 10.06%. Although there does not appear to be a systematic bias in the % (not absolute) errors, the errors tend to be larger for systems involving lower utilizations. For example, the system with all three locks having a utilization of .320 has an error of 77.4%. However, systems consisting of low-utilization locks account for a significantly smaller amount of delay. This observation may be illustrated by computing the absolute value of the deviations for each system. The total of the absolute value of deviations for all systems is only 7.24% of the total simulated delay for all systems.

10. Observations Concerning Lock Interdependence

The generalized model for lock delay interdependence may now be utilized to explore the nature of lock interdependence which ultimately would reveal the impacts of interdependence on the benefits associated with lock capacity improvements. This may be done by 1) plotting the interdependence coefficient for different values of the relevant variables, e.g number of locks, distance, and capacity, and 2) performing a sensitivity analysis on the expressions for S/I.

First, a plot of S/I for both a two lock system and three lock system provides a first look at how the coefficient changes with an inclusion of an additional lock in the system. In Figure 6, S/I is plotted versus volume for a two lock system with each lock having a capacity of 18 tows/day, a variance of 1.2, and distance of 20 miles. Also plotted are curves representing the inclusion of a third lock which is identical in every respect except that its capacity is 18, 30, or 100. Note that the curves do not shift uniformly, but rather, the decrease in S/I associated with increasing the size of the cluster from two to three locks increases with volume. Also, as the capacity of the third lock increases, the change in S/I associated with it decreases.

Next, the same system of two locks is plotted with curves representing three different three-lock systems in Figure 7. The three lock systems differ in their distance to the second lock, which is 100, 20, or 5 miles, but are identical in all other respects. This plot reveals that the farther away the third lock is from the two lock system, the smaller its contribution to the total interdependence. However, even very large distances, e.g. 100 miles, show some change in interdependence at very high utilizations. It is clear that the addition of a third lock reduces the value of S/I and the size of that reduction depends on the values of various lock and system variables.

11. Summary and Conclusions

Inland waterway navigation continues to be a vital component to our nations freight transportation system. With many inland navigation locks in need of repair, it is important to have reliable and efficient means of modeling the delays at locks for evaluation and investment planning. This paper has shown the development of an approximation to microsimulation that allows for an evaluation of lock delays that incorporate interdependencies.

A simulation experiment was performed to obtain the data necessary to calibrate the first step in the metamodeling approach. The results of simulation for both the system and isolated cases as well as S/I have been presented. It has been shown that the model of the factor variables for S/I adequately fits the data obtained from the simulation model. The metamodel was expanded to a system of iterative equations to incorporate systems of more than two locks. A validation of the lock coupling technique showed that the deviation from simulation for three lock systems averaged 10.1% for systems involving utilizations ranging from .320 to .890.

Using the metamodel for S/I, some observations concerning lock interdependence were made. It was found that the interdependence of the system increases with system size. The size of the increase depends on various lock characteristics. In addition to discovering some aspects of lock interdependence, the model may be used as part of an overall planning framework that includes project sequencing and scheduling.

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Appendix II. Notation

The following symbols are used in this paper:

a,b,c	Parameters for estimation
D_{ij}	Distance between locks i and j
I_i	Total isolated delay for lock i
n	Number of locks in a given system
S	Total system delay
S/I	Ratio of system to isolated delay (interdependence coefficient)
U	Relative utilization ρ/ρ_c
V/C	Volume to capacity ratio
W	Average wait time at locks with an M/G/1 queuing process
W_I	Estimated wait time for isolated locks
Y_{ij}	Amount of interdependence between locks i and j
Y_{ej}	Amount of interdependence between the set of locks e and lock j
$\alpha,\beta,\gamma,\delta$	Parameters for estimation
λ	Mean arrival time
μ	Mean service time
ρ	Lock utilization
ρ_{ej}	Effective lock utilization for the first j locks in the system
ρ_c	Maximum of "critical" utilization in the system
σ	Standard deviation of service times
σ_{ej}	Standard deviation of service times for first j locks in the system

Appendix III. Simulation Results for Three Lock Systems

Volume level, λ = 30 tows/day
Distance = 20 miles
ρ - utilization
σ^2 - variance of lock service time
I - computed isolated delay
$(S/I)_3$ - interdependence coefficient for the three locks
TW_s - total delay obtained from simulation
TW_m - total delay obtained from the meta model

Lock	ρ	W	σ^2	I	$(S/I)_3$	TM_s	TM_m	%ERR
1	0.890	24.349	1.560	34.25				
2	0.890	22.454	1.561	34.27				
3	0.890	24.443	1.552	33.99	0.751	71.246	76.990	8.06
1	0.890	25.801	1.560	34.25				
2	0.750	2.477	1.113	4.27				
3	0.890	25.330	1.552	33.99	0.770	53.608	55.853	4.19
1	0.890	25.742	1.560	34.24				
2	0.660	0.900	0.784	1.42				
3	0.890	25.276	1.552	33.99	0.781	51.918	54.396	4.77
1	0.890	26.825	1.560	34.24				
2	0.320	0.199	0.213	0.06				
3	0.890	26.098	1.552	33.99	0.823	53.122	56.202	5.80
1	0.750	2.846	1.113	4.27				
2	0.890	23.750	1.561	34.27				
3	0.890	25.205	1.552	33.99	0.770	51.802	55.863	7.84
1	0.750	3.018	1.113	4.27				
2	0.750	2.612	1.113	4.27				
3	0.890	26.497	1.551	33.98	0.802	32.127	34.098	6.13
1	0.750	3.167	1.113	4.27				
2	0.660	0.994	0.784	1.42				
3	0.890	26.042	1.551	33.98	0.813	30.204	32.230	6.71
1	0.750	3.030	1.113	4.27				
2	0.320	0.227	0.213	0.06				
3	0.890	27.050	1.551	33.98	0.844	30.307	32.316	6.63
1	0.750	2.547	1.113	4.27				
2	0.890	31.306	1.561	34.27				
3	0.750	2.961	1.107	4.24	0.790	36.814	35.586	3.33

Lock	ρ	W	σ^2	I	$(S/I)_3$	TM_s	TM_m	%ERR
1	0.750	3.463	1.113	4.27				
2	0.750	2.903	1.113	4.27				
3	0.750	3.171	1.107	4.24	0.820	9.538	10.474	9.82
1	0.750	3.434	1.113	4.27				
2	0.660	0.902	0.784	1.42				
3	0.750	3.214	1.106	4.23	0.830	7.550	8.238	9.11
1	0.750	3.547	1.113	4.27				
2	0.320	0.190	0.213	0.06				
3	0.750	3.218	1.106	4.23	0.862	6.955	7.375	6.05
1	0.660	0.869	0.784	1.42				
2	0.890	25.865	1.561	34.27				
3	0.890	25.595	1.551	33.98	0.781	52.329	54.407	3.97
1	0.660	0.938	0.784	1.42				
2	0.750	2.956	1.113	4.27				
3	0.890	26.622	1.551	33.98	0.813	30.516	32.232	5.62
1	0.660	0.992	0.784	1.42				
2	0.660	1.131	0.784	1.42				
3	0.890	27.017	1.551	33.98	0.828	29.140	30.490	4.63
1	0.660	1.010	0.784	1.42				
2	0.320	0.223	0.213	0.06				
3	0.890	27.730	1.551	33.98	0.856	28.963	30.345	4.77
1	0.660	0.939	0.784	1.42				
2	0.890	33.315	1.561	34.27				
3	0.750	3.204	1.107	4.24	0.800	37.458	33.091	11.66
1	0.660	1.185	0.784	1.42				
2	0.750	3.530	1.113	4.27				
3	0.750	3.318	1.106	4.23	0.830	8.033	8.240	2.58
1	0.660	1.289	0.784	1.42				
2	0.660	0.938	0.784	1.42				
3	0.750	3.303	1.106	4.23	0.846	5.530	5.989	8.31
1	0.660	1.289	0.784	1.42				
2	0.320	0.236	0.213	0.06				
3	0.750	3.368	1.106	4.23	0.874	4.892	4.993	2.06
1	0.660	1.152	0.784	1.42				
2	0.890	29.154	1.561	34.27				
3	0.660	0.971	0.784	1.42	0.813	31.277	34.875	11.51

Lock	ρ	W	σ^2	I	$(S/I)_3$	TM_s	TM_m	%ERR
1	0.660	1.422	0.784	1.42				
2	0.750	2.801	1.113	4.27				
3	0.660	0.993	0.784	1.42	0.842	5.216	5.988	14.80
1	0.660	1.486	0.784	1.42				
2	0.660	0.811	0.784	1.42				
3	0.660	1.025	0.784	1.42	0.857	3.321	3.658	10.12
1	0.660	1.505	0.784	1.42				
2	0.320	0.165	0.213	0.06				
3	0.660	1.169	0.784	1.42	0.885	2.839	2.568	9.54
1	0.320	0.222	0.213	0.06				
2	0.890	26.768	1.561	34.27				
3	0.890	27.338	1.551	33.98	0.823	54.328	56.215	3.47
1	0.320	0.226	0.213	0.06				
2	0.750	2.732	1.113	4.27				
3	0.890	26.044	1.551	33.98	0.844	29.002	32.318	11.43
1	0.320	0.256	0.213	0.06				
2	0.660	1.215	0.784	1.42				
3	0.890	26.814	1.551	33.98	0.856	28.285	30.345	7.28
1	0.320	0.259	0.213	0.06				
2	0.320	0.267	0.213	0.06				
3	0.890	28.284	1.551	33.98	0.890	28.809	30.348	5.34
1	0.320	0.195	0.213	0.06				
2	0.890	33.286	1.561	34.27				
3	0.750	3.442	1.107	4.24	0.843	36.922	33.079	10.41
1	0.320	0.209	0.213	0.06				
2	0.750	3.751	1.113	4.27				
3	0.750	3.461	1.106	4.23	0.862	7.421	7.378	0.59
1	0.320	0.216	0.213	0.06				
2	0.660	0.976	0.784	1.42				
3	0.750	3.448	1.106	4.23	0.874	4.640	4.993	7.60
1	0.320	0.225	0.213	0.06				
2	0.320	0.300	0.213	0.06				
3	0.750	3.848	1.106	4.23	0.913	4.373	3.969	9.23
1	0.320	0.190	0.213	0.06				
2	0.890	31.413	1.561	34.27				
3	0.660	1.026	0.784	1.42	0.856	32.629	34.993	7.25

Lock	ρ	W	σ^2	I	$(S/I)_3$	TM_s	TM_m	%ERR
1	0.320	0.246	0.213	0.06				
2	0.750	2.859	1.113	4.27				
3	0.660	1.202	0.784	1.42	0.874	4.306	5.023	16.64
1	0.320	0.249	0.213	0.06				
2	0.660	0.913	0.784	1.42				
3	0.660	1.210	0.784	1.42	0.885	2.372	2.568	8.29
1	0.320	0.246	0.213	0.06				
2	0.320	0.181	0.213	0.06				
3	0.660	1.215	0.784	1.42	0.922	1.641	1.417	13.70
1	0.320	0.175	0.213	0.06				
2	0.890	27.238	1.561	34.27				
3	0.320	0.247	0.212	0.06	0.905	27.660	34.464	24.60
1	0.320	0.256	0.213	0.06				
2	0.750	2.974	1.113	4.27				
3	0.320	0.250	0.212	0.06	0.919	3.480	4.326	24.30
1	0.320	0.275	0.213	0.06				
2	0.660	1.084	0.784	1.42				
3	0.320	0.200	0.212	0.06	0.928	1.559	1.449	7.04
1	0.320	0.277	0.213	0.06				
2	0.320	0.246	0.213	0.06				
3	0.320	0.202	0.212	0.06	0.956	0.725	0.164	77.36

Table 1 Values of Factor Variables Used in Simulation Experiment

	Linehaul Distance (Miles)	Critical Utilization	Utilization Ratio
Level 1	5	.320	.053
Level 2	20	.660	.369
Level 3	30	.750	.633
Level 4	80	.890	.845
Level 5		1.000	

Table 2 Summary of Volume and Utilizations for Lock 2

U \ $\rho c1$	0.890		0.750		0.660		0.320	
	$\lambda 2$	$\rho 2$	$\lambda 2$	$\rho 2$	$\lambda 2$	$\rho 2$	$\lambda 2$	$\rho 2$
1.000	53.93	.890	45.45	.750	40.00	.660	19.39	.320
0.845	45.45	.750	38.36	.633	33.94	.560	16.36	.270
0.633	33.94	.560	28.78	.475	25.33	.418	12.24	.202
0.369	20.00	.330	16.36	.270	14.54	.240	7.27	.120
0.053	2.85	.047	2.18	.039	2.12	.035	1.03	.017

Table 3 Results of Simulation for U=1.00 and ρ_C=.89

U = 1.00

Lock 1: V/C = 0.89
Lock 2: V/C = 0.89

		Lock 1		Lock 2				
		Mean I	Mean S	Mean I	Mean S	Tot. I	Tot. S	S/I
	Dir 1	47.184	33.362	47.184	34.336			
5	Dir 2	46.856	33.337	46.856	34.006	94.04	67.52	0.7180
	Dir 1	47.184	38.008	47.184	37.824			
20	Dir 2	46.856	37.414	46.856	41.562	94.04	77.40	0.8231
	Dir 1	47.184	39.477	47.184	40.046			
30	Dir 2	46.856	40.721	46.856	38.928	94.04	79.59	0.8463
	Dir 1	47.184	43.751	47.184	44.038			
80	Dir 2	46.856	45.177	46.856	45.091	94.04	89.03	0.9467

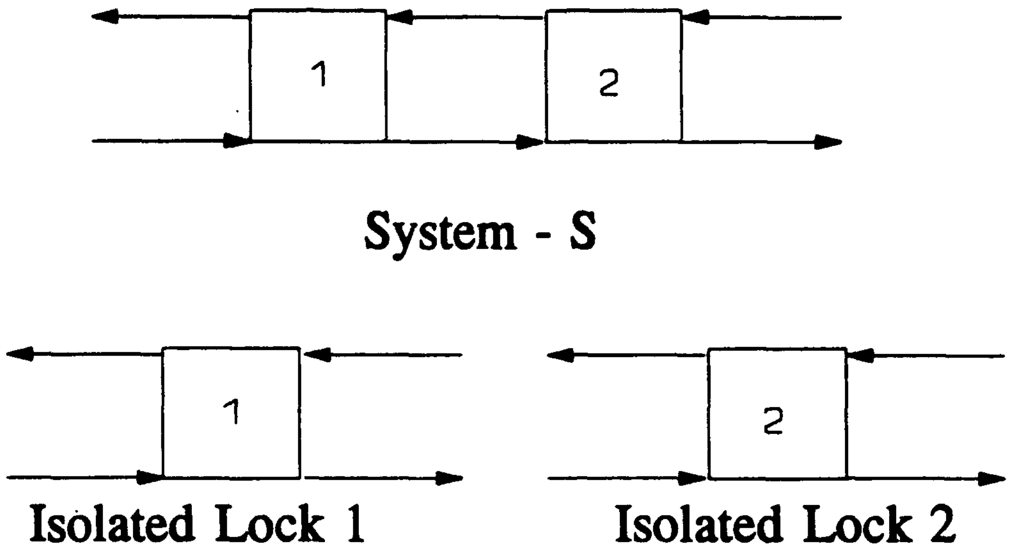

Figure 1 System and Isolated Configurations for Simulation

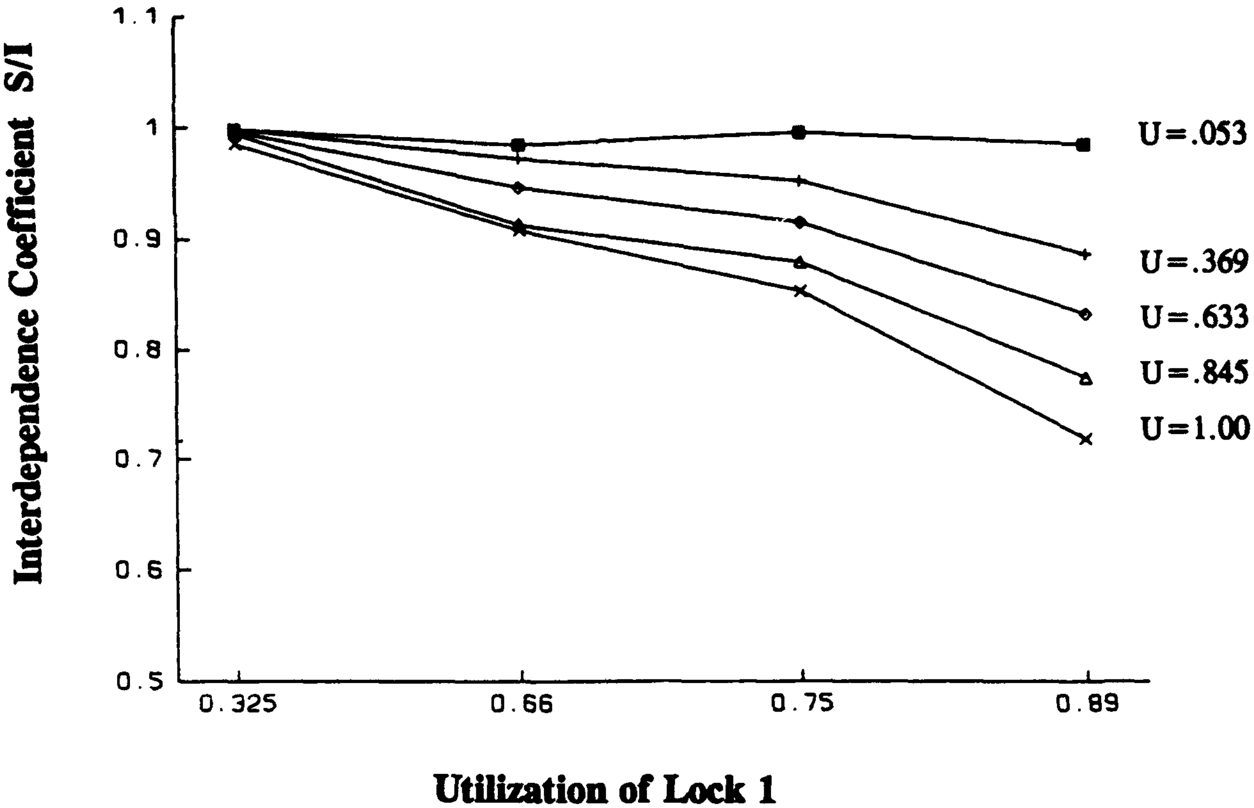

Figure 2 Simulation Results for D=5

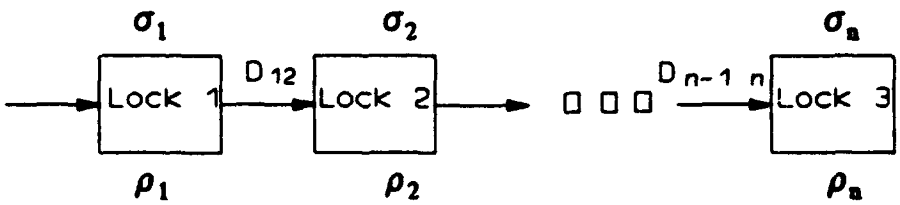

Figure 3 Series of n Interdependent Locks

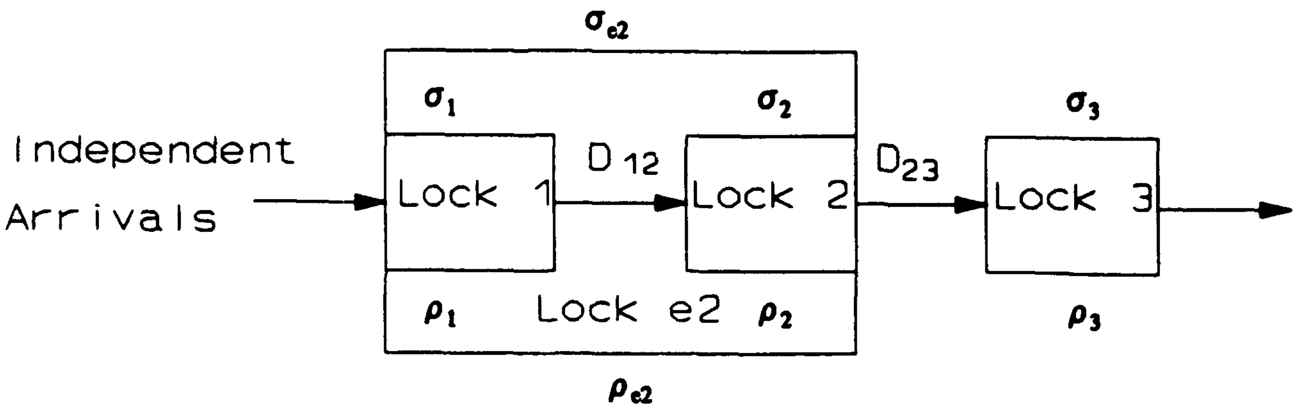

Figure 4 Conceptual System for Lock Coupling

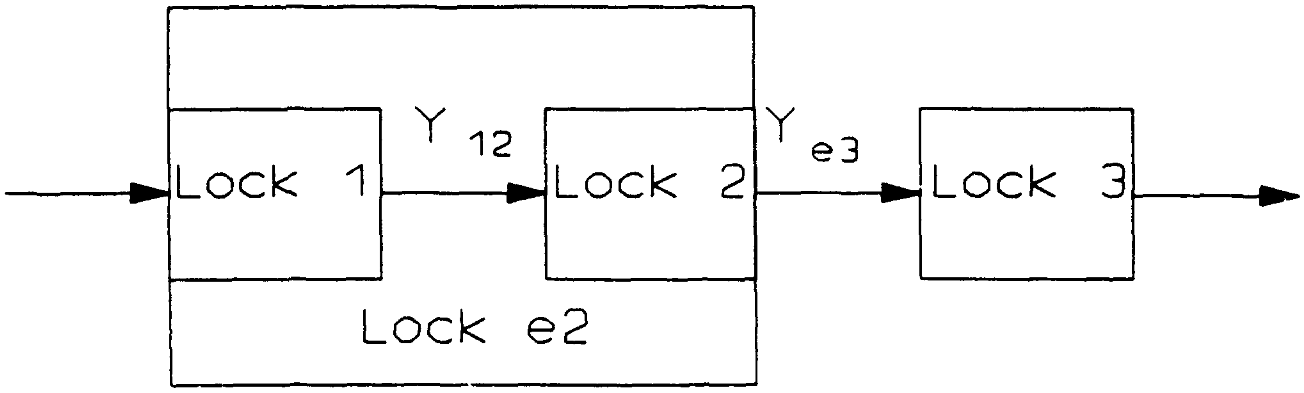

Figure 5 Adding Interdependence Variables for a 3 Lock System

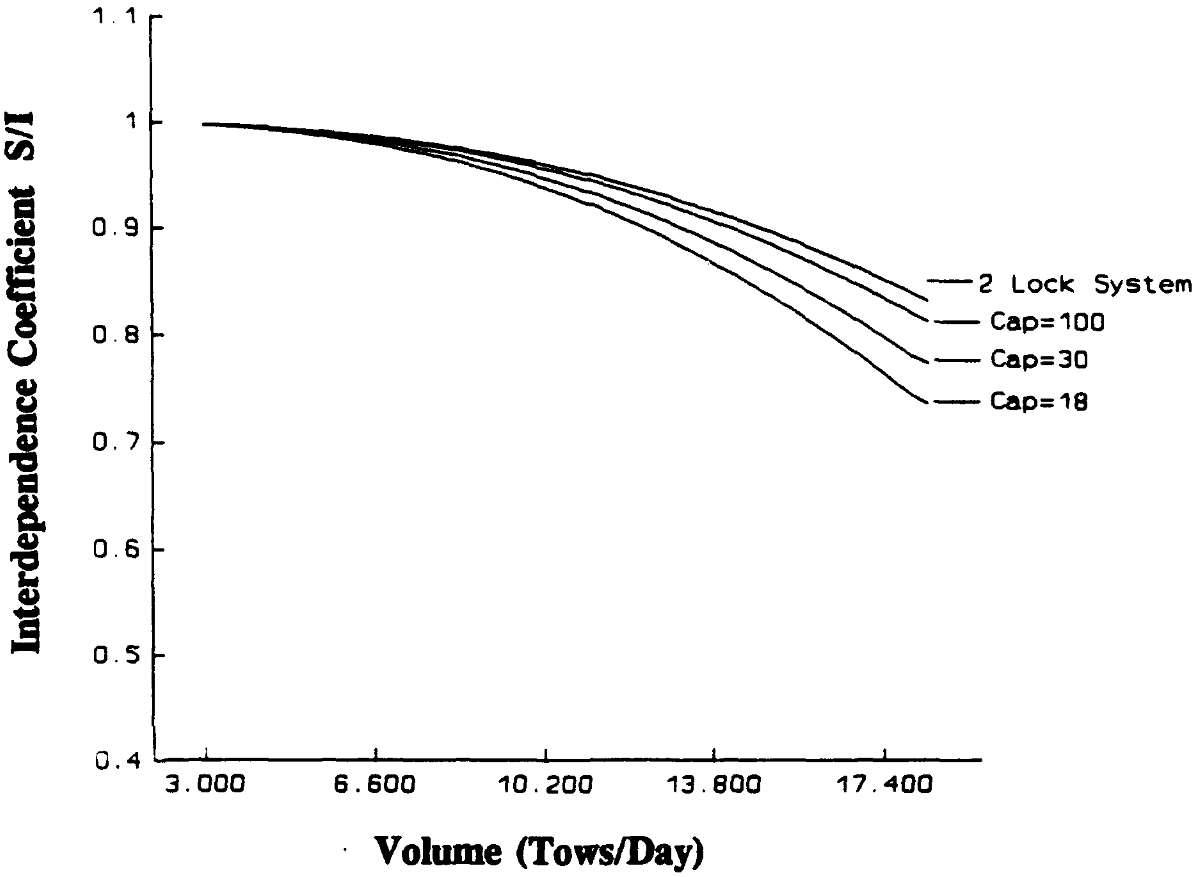

Figure 6 S/I for a 2 Lock and 3 Lock Systems of Capacity 100, 30 and 18

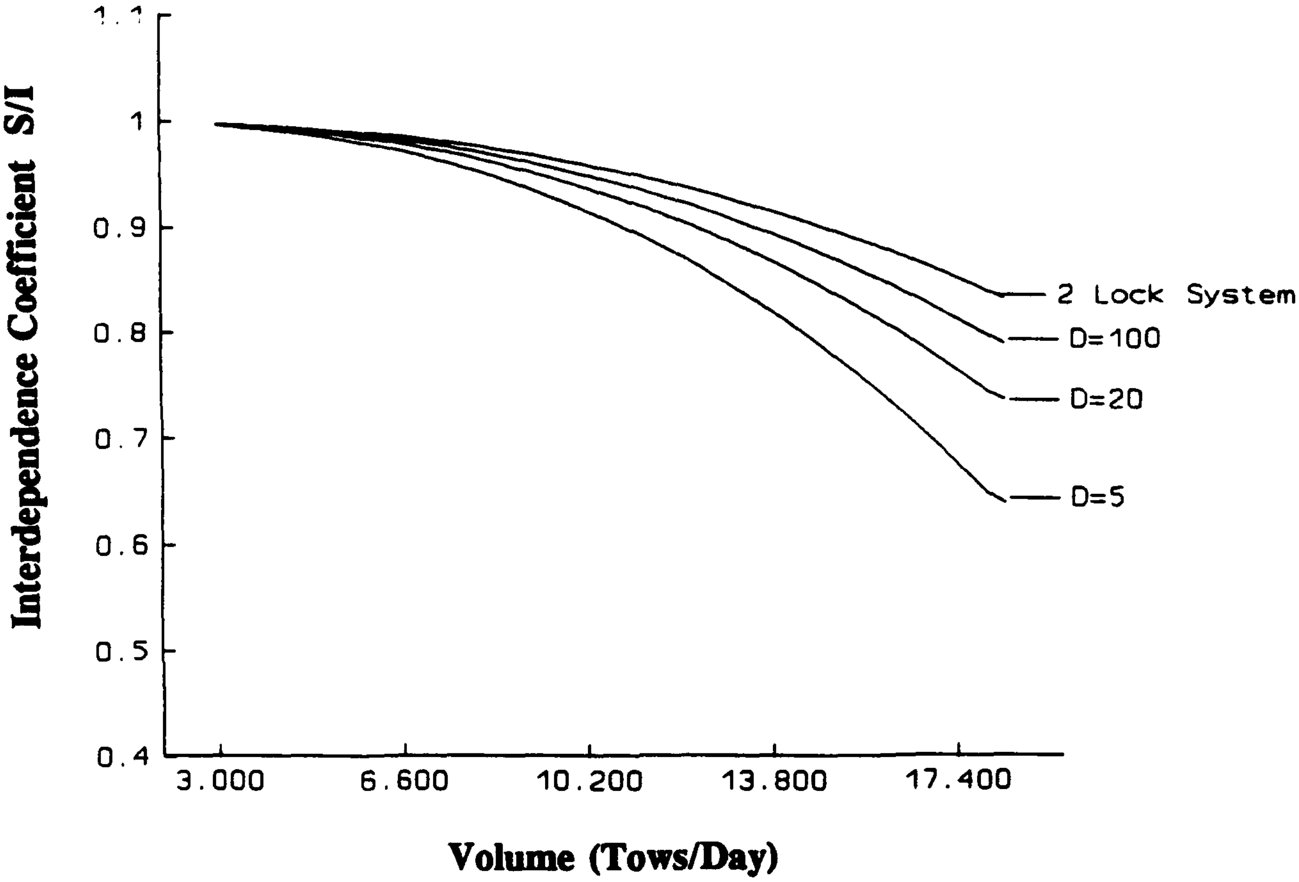

Figure 7 S/I for a 2 Lock and 3 Lock Systems of Distance 100, 20, and 5

APPROXIMATE DELAYS DUE TO LOCK SERVICE INTERRUPTIONS

by

Venkatesh Ramanathan and Paul Schonfeld

Approximate Delays Due to Lock Service Interruptions

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ABSTRACT

A model is developed in the form of one relatively simple equation to estimate tow delays due to one lock service interruption. This model is developed by combining deterministic queuing theory and an adjustment factor estimated statistically from simulation results. The model provides accurate estimates of delays more quickly and inexpensively than simulation

1.INTRODUCTION

A model is developed to estimate the delay due a single lock service interruption in a waterway lock. The existing simulation model[1,2] was modified to estimate the actual delay due a single stall. This delay can now be estimated quite accurately with the proposed model. The model combines non-probabilistic queuing relations based on uniform continuous arrival and service rates with a factor derived from simulation results that accounts for discrete probabilistic arrivals and service times.

2. PROPOSED MODEL

The effect of a single stall based on uniform continuous flow is shown in Fig.1. The service interruption would reduce the normal capacity c to a partial capacity p in a lock with multiple chambers. If the lock has a single chamber, the partial capacity will probably drop to zero, in which case, the equation becomes even simpler.

Let:

c = normal lock capacity (tows/ hr)

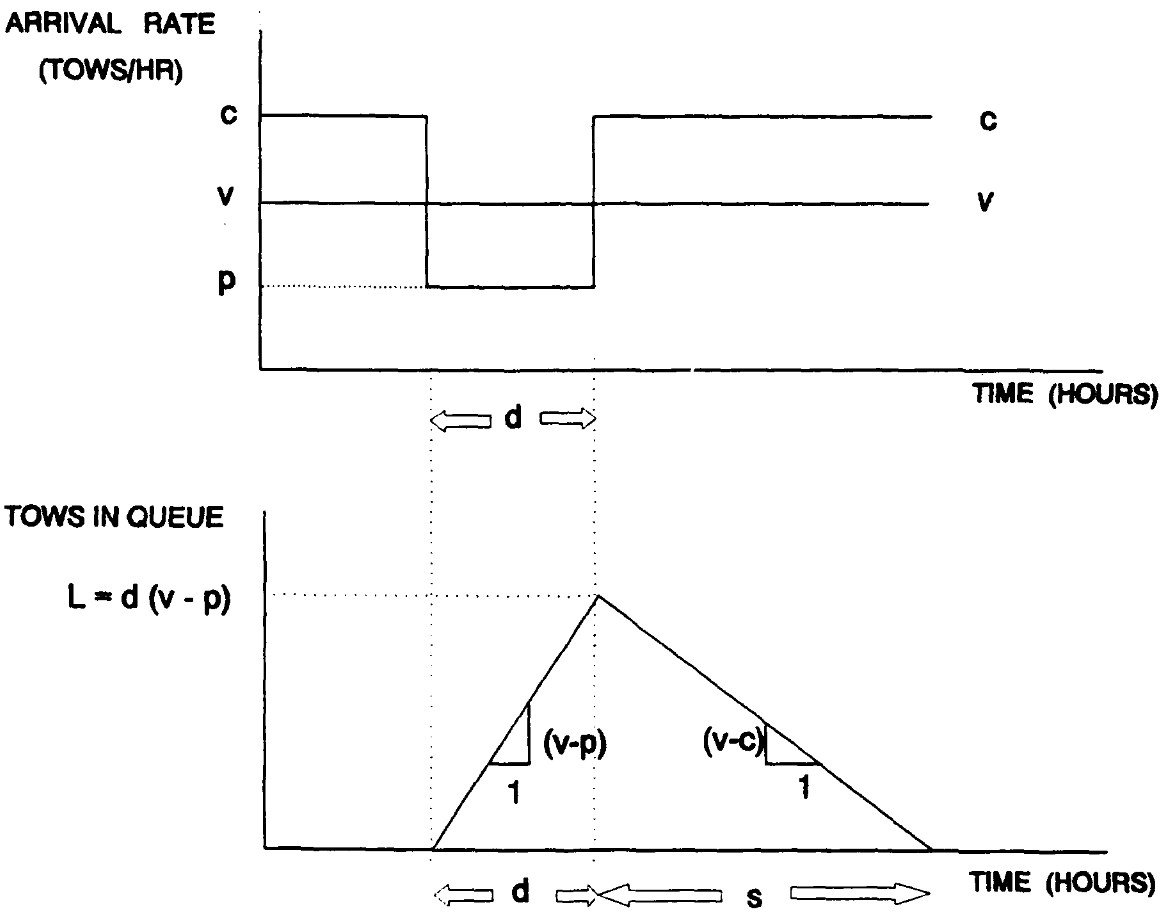
p = partial lock capacity (tows/ hr)

d = duration of partial capacity condition or stall = queue growth time (hours)

v = traffic volume = tow arrival rate from both directions (tows / hr)

s = queue dissipation time (hours)

FIG 1 EFFECT OF ONE STALL BASED ON UNIFORM CONTINUOUS FLOW



D_d= deterministic delay with continuous flow (tow hours)

L = maximum queue length (tows)

The queue dissipation time s is given by:

$$s = \frac{d(v - p)}{-(v - c)} = \frac{d(v - p)}{(c - v)} \tag{1}$$

The delay D_d for the tows is:

$$D_d = \frac{L(d + s)}{2} = \left(\frac{d(v - p)}{2}\right)\left(\frac{d(v - p)}{(c - v)} + d\right) \tag{2}$$

If the partial capacity p is zero, then the delay D_d in equation 2 simplifies to:

$$D_d = \frac{d\,v}{2}\left(\frac{d\,v}{(c - v)} + d\right) \tag{3}$$

Example:

If v = 30 tows/hr, c = 40 tows/hr, d = 2 hours and p = 1 tow/hr, then the delay D_d will be, using equation 2:

$$D_d = \left(\frac{2(3 - 1)}{2}\right)\left(\frac{2(3 - 1)}{(4 - 3)} + 2\right) = 12 \textit{ tow hours}$$

The Eqs. 2 and 3 are general enough to apply to one or two way traffic and to single or multiple chamber locks. Since their derivation is based on uniform continuous arrival rates and service rates, they are only approximations for a real waterway with probabilistic arrivals and service times.

3. SIMULATION RESULTS

The simulation model was modified to estimate the delay due to a single stall. The delay due to a single stall is the difference between the delay with a stall and the delay without a stall. The delay due to stall increases for the duration of the stall and then decreases and finally becomes negligible. The results were recorded for a sufficiently long period to insure that the full effects of the stall were captured. The results from the simulation experiment were then compared with those obtained using Eq.3.

The comparison was conducted for a variety of volume/capacity ratios ranging from 0.4 to 0.95. To reduce the variance of the delay, the final result used for comparison was obtained by averaging the output from 40 independent simulation runs. To insure that the systems reach steady states before the stall occurs, the first 12,000 tow waiting times are discarded from each simulation run.

The results are shown in Table 1. Fig.2 shows that the discrepancy between the simulated and theoretical delay approaches zero as the stall duration increases. Fig 3 shows that similar results are obtained as the delay increases. Fig.3 is consistent with Fig.2, if we remember that delay D_d varies roughly with the square of stall duration d in Eq.3.

TABLE 1 COMPARISON OF SIMULATED DELAY WITH DETERMINISTIC QUEUING DELAY

a) $\mu = 25 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.4$

Stall Dur (Days)	Delay(D_s) (Simulated) (tow-days)	Delay(D_d) (Determ.) (tow-days)	Discrepancy(%) $\frac{(D_s-D_d)}{D_d}$	Std. Dev. (Simul)	Std. Error (Simul)	t-test Value
0.3	0.985	0.75	31.3	0.41	0.07	3.625
1	9.597	8.33	15.2	2.25	0.36	3.560
2	36.630	33.33	9.9	9.31	1.47	2.242
3	79.740	75.00	6.3	12.34	1.95	2.430
4	138.400	133.33	3.8	21.26	3.36	1.508
6	306.160	300.00	2.0	39.85	6.30	0.978
8	541.920	533.33	1.6	71.21	11.26	0.762
10	843.500	833.33	1.2	86.50	13.68	0.745
12	1208.400	1200.00	0.7	116.23	18.38	0.457

b) $\mu = 16.67 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.6$

Stall Dur (Days)	Delay(D_s) (Simulated) (tow-days)	Delay(D_d) (Determ.) (tow-days)	Discrepancy(%) $\frac{(D_s-D_d)}{D_d}$	Std. Dev. (Simul)	Std. Error (Simul)	t-test Value
0.3	1.556	1.125	38.3	0.71	0.11	3.85
1	15.198	12.500	21.6	6.88	1.09	2.48
2	57.390	50.000	14.8	15.75	2.49	2.96
3	123.870	112.500	10.1	19.61	3.10	3.69
4	213.470	200.000	6.7	33.46	5.29	2.55
6	465.890	450.000	3.5	63.15	9.98	1.59
8	818.600	800.000	2.3	86.42	13.66	1.36
10	1269.050	1250.000	1.5	92.10	14.56	1.30
12	1819.600	1800.000	1.0	139.96	22.12	0.89

TABLE 1 COMPARISON OF SIMULATED DELAY WITH DETERMINISTIC QUEUING DELAY

c) $\mu = 12.5 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.8$

Stall Dur (Days)	Delay(D_s) (Simulated) (tow-days)	Delay(D_d) (Determ.) (tow-days)	Discrepancy(%) $\frac{(D_s-D_d)}{D_d}$	Std. Dev. (Simul)	Std. Error (Simul)	t-test Value
0.3	3.253	2.25	44.5	1.71	0.27	3.70
1	31.920	25.00	27.7	12.28	1.94	3.56
2	117.890	100.00	17.9	28.06	4.44	4.07
3	252.950	225.00	12.4	51.21	8.09	3.45
4	437.900	400.00	9.4	69.29	10.95	3.45
6	956.100	900.00	6.2	110.40	17.46	3.21
8	1672.110	1600.00	4.5	199.29	31.51	2.28
10	2586.610	2500.00	3.4	300.90	47.58	1.82
12	3682.85	3600.00	2.3	348.40	55.09	1.50

d) $\mu = 10.53 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.95$

Stall Dur (Days)	Delay(D_s) (Simulated) (tow-days)	Delay(D_d) (Determ) (tow-days)	Discrepancy(%) $\frac{(D_s-D_d)}{D_d}$	Std. Dev. (Simul)	Std. Error (Simul)	t-test Value
0.3	13.29	9	47.7	5.21	0.82	5.20
1	132.06	100	32.0	41.56	6.57	4.87
2	489.25	400	22.2	126.30	19.97	4.46
3	1044.00	900	16.0	135.23	21.39	6.73
4	1787.59	1600	11.7	283.29	44.79	4.18
6	3902.84	3600	8.4	509.11	80.50	3.75
8	6798.16	6400	6.2	736.77	116.49	3.42
10	10493.12	10000	4.9	922.00	126.81	3.38
12	14907.31	14400	3.5	1196.53	189.19	2.68

FIG 2 VARIATION OF STOCHASTIC DELAY ADJUSTMENT FACTOR WITH STALL DURATION

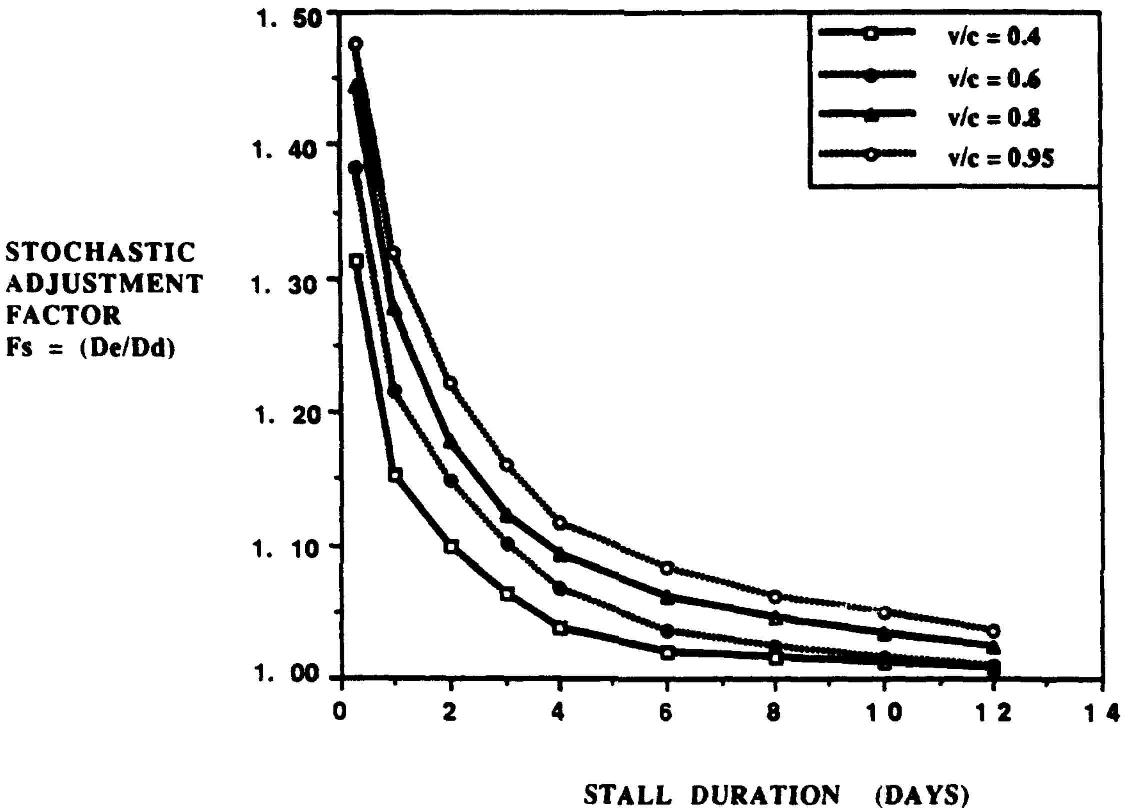
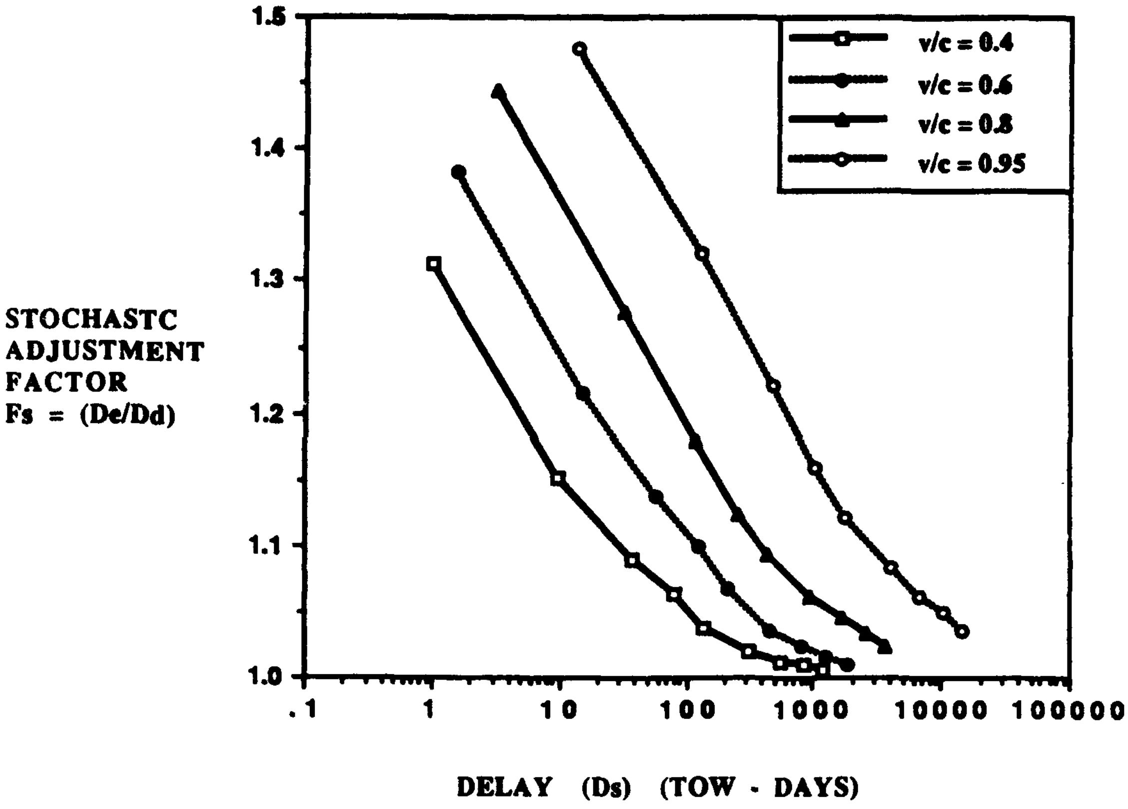


FIG 3 VARIATION OF STOCHASTIC DELAY ADJUSTMENT FACTOR WITH SIMULATED DELAY



4. STOCHASTIC DELAY ADJUSTMENT FACTOR

A stochastic adjustment factor (F_s) was derived which, when multiplied with the results of the Eq.3, produces results approximating the simulation output. This factor was estimated statistically, using simulation results as data.

Let:

D_d = deterministic delay with continuous flow

D_s = simulated delay

D_e = estimated delay $= F_s D_d$

F_s = stochastic adjustment factor $= \dfrac{D_e}{D_d}$

Then:

$$F_s = \frac{D_s}{D_d} \tag{4}$$

The stochastic adjustment factor which accounts for the probabilistic arrivals and service times decreases with the stall duration and increases with the volume/capacity ratio(Fig.2). The factor is large for smaller stall durations and negligible at longer stalls. When the arrival rate is large, the factor is very small even at smaller stall durations.

Eq.4 can be rearranged as :

$$D_s = [F_s]\ [D_d]$$

Substituting the statistically estimated relation for the stochastic adjustment factor F_s and Eq.2 for D_d we obtain the following:

$$D_s = \left[1 + \left(0.6\left(\frac{v}{c}\right)e^{-0.432\,d}\right)\right]\left[\left(\frac{d(v-p)}{2}\right)\left(\frac{d(v-p)}{(c-v)} + d\right)\right], \qquad R^2=0.93 \tag{5}$$

A comparison of the simulated delay D_s and estimated delay D_e is given in Table 2. It is observed that the deviation between the simulated delay and the estimated delay is less than 2% for larger stall durations when the volume/capacity ratios are less than 0.6. The deviation is also below 2% for stall durations less than 4 days when the volume/capacity ratios are above 0.6.

5. CONCLUSION

The model developed in this paper provides a good approximation for the estimation of delay due to a single stall in a real waterway with probabilistic arrivals and service times. The deviation between the estimated delay and the simulated delay is less than 2% for larger stall durations when the volume/capacity ratio is small and for smaller stall durations when the volume/capacity ratio is large. Thus, it becomes a very fast and inexpensive substitute for simulation.

TABLE 2 COMPARISON OF SIMULATED DELAY WITH ESTIMATED DELAY

$a)\ \mu = 25 tows/day\ ;\ \lambda = 10 tows/day\ ;\ V/C = 0.4$

Stall Duration (d) (Days)	Simulated Delay (D_s) (tow-days)	Estimated Delay (D_e) (tow-days)	% Deviation 100 x (D_e-D_s)/D_s
0.3	0.985	0.907	7.91
1	9.597	9.621	-2.50
2	36.630	36.696	-1.80
3	79.740	79.875	-1.69
4	138.400	139.009	-0.43
6	306.160	305.370	0.26
8	541.920	537.369	0.80
10	843.500	835.829	0.90
12	1208.400	1201.614	0.56

$b)\ \mu = 16.67 tows/day\ ;\ \lambda = 10 tows/day\ ;\ V/C = 0.6$

Stall Duration (d) (Days)	Simulated Delay (D_s) (tow-days)	Estimated Delay (D_e) (tow-days)	% Deviation 100 x (D_e-D_s)/D_s
0.3	1.556	1.480	4.88
1	15.198	15.412	-1.40
2	57.390	57.850	0.80
3	123.870	123.581	0.23
4	213.470	212.780	0.32
6	465.890	462.105	0.80
8	818.600	808.800	1.19
10	1269.050	1255.875	1.03
12	1819.600	1803.600	0.87

TABLE 2 COMPARISON OF SIMULATED DELAY WITH ESTIMATED DELAY

c) $\mu = 12.5 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.8$

Stall Duration (d) (Days)	Simulated Delay (D_s) (tow-days)	Estimated Delay (D_e) (tow-days)	% Deviation 100 x $(D_e-D_s)/D_s$
0.3	3.253	3.195	1.78
1	31.920	32.775	-2.60
2	117.890	120.200	1.95
3	252.950	254.540	-0.60
4	437.900	434.000	0.89
6	956.100	931.500	2.57
8	1672.110	1624.000	2.87
10	2586.610	2515.959	2.73
12	3682.850	3609.684	2.01

d) $\mu = 10.5 tows/day$; $\lambda = 10 tows/day$; $V/C = 0.95$

Stall Duration (d) (Days)	Simulated Delay (D_s) (tow-days)	Estimated Delay (D_e) (tow-days)	% Deviation 100 x $(D_e-D_s)/D_s$
0.3	13.290	13.500	1.58
1	132.060	137.000	-3.74
2	489.250	496.000	-1.36
3	1044.000	1039.500	0.40
4	1787.590	1761.610	1.45
6	3902.840	3753.360	3.80
8	6798.160	6514.560	4.17
10	10493.120	10075.000	3.98
12	14907.310	14445.000	3.09

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COMPUTATIONAL CHARACTERISTICS OF A NUMERICAL MODEL FOR SERIES OF WATERWAY QUEUES

by

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ABSTRACT

A numerical method has been developed for estimating delays on congested waterways represented by series of G/G/1 queues, i.e., with generally distributed arrival and service times and one chamber per lock. It is based on a metamodelling approach which develops simple formulas to approximate the results of simulation models. The functional form of the metamodels is derived from queueing theory while their coefficients are statistically estimated from simulation results. The algorithm scans along a waterway and sequentially estimates at each lock the arrival distributions, departure distributions, and delays. It can be applied to systems with two-way traffic through common bi-directional servers as well as to one-way traffic systems.

Computational results are presented in this paper to illustrate the speed and convergence properties of the algorithm and to investigate some of its variants. The algorithm works satisfactorily and flexibly with different convergence criteria and scanning processes. For an illustrative 20-lock system, parameter estimates converge within five iterations and less than three seconds of CPU time to differences lower than 0.1 percent between successive iterations. The computation time is found to increase only linearly with the number of locks in the system, thus allowing the analysis of very large systems of interdependent queues.

INTRODUCTION

Inland waterway transportation is quite important in the U.S. and elsewhere, especially for heavy or bulky commodities, since it is inexpensive, energy efficient and safe. Most U.S. waterways consist of stepped navigable pools formed by dams across natural rivers. The lock structures used to raise or lower vessels between adjacent pools constitute the major bottlenecks in the waterway network (*1*) and generate extensive queues. Some locks have only one chamber, while others may have two parallel chambers whose characteristics may differ. The service time distributions at locks depend heavily on chamber size and tow size distributions. The lock service time distributions would be affected by the chamber assignment discipline at locks with two dissimilar chambers.

The waterway locks constitute a series of queueing stations. In queueing terms, locks are the servers and tows are customers waiting to be served by locks. Tows from both directions, upstream and downstream, share the same lock servers while in most other queueing systems servers are exclusively one-directional. Hence, the term "two-way traffic operations" characterizes the lock system analyzed below.

Arrival and service time distributions at locks are fairly complex. Carroll (*2*) and Desai (*3*) found that service times are not exponentially distributed, and arrivals are not Poisson distributed. Other standard distributions have been tested for the present study, without consistent success. Thus, empirical distributions (specified for 50 intervals) are used here for simulation while general tabular distributions, described usually only by their means and variances, are used for queueing models. Although locks with a single chamber may be modeled as G/G/1 queueing systems (i.e., "general arrival/general service times/1 server per station"), locks with two parallel chambers may not be treated simply as G/G/2 queueing systems unless these chambers are identical.

Considerable interdependence may exist among locks in a series. The departure distributions differ from the arrival distributions since the service time distributions change the tow headways. Departures from one lock usually affect arrivals at the next lock. Interdependence among locks increases the difficulty in estimating systemwide delays since the interarrival time distributions from adjacent locks must be identified at each lock. Two-way traffic operation through common servers complicates the interdependence of lock delays and precludes the use of some otherwise interesting queueing models.

Random failures (called "stalls") contribute significantly to the difficulties in estimating delays. Stalls, which interrupt lock operations and thereby increase delays, are relatively rare compared to other events and quite difficult to predict. Thus, Kelejian's efforts to model stall frequencies and durations have not yet yielded strong results despite the rigorous statistical methods employed (*4*).

The following special problems are encountered in estimating delays of waterway queues:

1. Arrival and service time distributions are too complex for analytic solutions and do not match known statistical distributions.
2. Parallel chambers are not identical.
3. Service time distributions are affected by the chamber assignment discipline.
4. Considerable interdependence exists among a series of locks.
5. Two-way traffic operates through bi-directional chambers.
6. Arrival distributions depend on distances and speed distributions between locks, as well as

departures from adjacent locks.

7. Stalls increase the means and variances of delays.

Delay estimation for a realistic lock queueing system has been undertaken by Dai and Schonfeld (*5,6,7*) using several approaches, including queueing theory, simulation, and numerical methods. Their simulation model does deal with all seven problems listed above and is quite efficient for analyzing particular system configurations. However, when large numbers of system alternatives must be evaluated for investment scheduling, a much faster numerical method, which approximates the results of simulations, becomes preferable. The primary purpose of this paper is to assess the computational characteristics of the numerical method developed for this role. In particular, the number of iterations and the computation time required to reach convergence using various criteria and scanning procedures are investigated. The effects of system size (i.e., number of locks) on computational requirements are also examined.

LITERATURE REVIEW

The available analytic solutions for estimating delays in G/G/1 queues are quite inadequate. Kleinrock (*8*) suggested an approximation solution for a G/G/1 queue with heavy traffic, which is a useful upper bound for average waiting times in G/G/1 queues. Bertsimas (*9*) derived an exact solution for mixed generalized Erlang distributed arrivals and service times. However, without a departure function this result is difficult to extend to a series of locks.

Exact solutions for networks of queues are still limited to Markovian networks. For more general networks of queues, approximation methods are employed in Whitt (*10*) and Albin (*11*) for system performance analysis. The underlying concept is to decompose the network into individual queues that are analyzed independently, and then recombine the results. Their efforts are quite valuable but employ unreasonable coefficients of variation (standard deviation divided by mean) and are not applicable to bi-directional servers.

System simulation models to analyze lock delays and tow travel times were developed by Howe (*12*) and by Carroll and Bronzini (*13*). These two models, which did not account for stalls, required considerable data and computer time. However, simulation models can, in principle, represent the complexities of traffic on waterway networks much better than analytic queueing models.

A new waterway simulation model was developed by Dai and Schonfeld (*5*). This model accommodates generally (i.e., arbitrarily) distributed trips and service times. It can also evaluate stall effects. This simulation model requires only a few seconds to a few minutes on a PS/2 computer for each run, depending on traffic volumes, simulation period durations, network size, etc. Still, it is hardly affordable for direct application in large combinatorial network investment problems.

To avoid the computational expense of simulation, a metamodelling approach (*14*) was developed. This approach consists of 1) developing and validating a simulation model to represent waterway networks with queues at locks, 2) formulating functions developed from queueing theory for delays through series of locks, 3) statistically estimating the parameters of these functions using simulation results, and 4) employing an iterative sequential scanning procedure to estimate interarrival and interdeparture time distributions lock by lock until results converge at each lock. Thus, relatively simple equations may serve as a proxy for the simulation model.

SIMULATION MODEL

The simulation model developed for this work is documented in Dai and Schonfeld (*5*). Only a brief description is provided below.

The simulation model was developed using the PMS (lock Performance Monitoring System) data base, which includes very detailed information on traffic through the locks as well as physical aspects of lockages (*15*). The simulation model is programmed in Fortran-77, which provides great flexibility in modelling. Basically, it is a stochastic, microscopic and event-scanning simulation model which can handle any distributions for trip generation, travel speeds, lock service times and tow sizes. Currently, tabular distributions based on empirical observations are used for most input variables. A FIFO (First-In-First-Out) service discipline is currently employed. This model simulates two-way traffic through common servers and accounts for stalls.

The validation results (*5,6,7*) show that the overall mechanism of the simulation model is correct, and that the simulated average waiting times for each lock and for the entire series of locks are closely similar to those observed. Dai (*6*) documents the statistical methods used in developing, validating, and applying the simulation model.

NUMERICAL METHOD

Overview

A numerical method has been developed for estimating delays through a series of queues with bi-directional servers. A brief description of the method follows, while details of its development and validation are provided elsewhere (*6,7*).

The method consists of three major modules, namely arrival processes, departure processes, and delay functions (as summarized in FIGURE 1), which are applied in that sequence at each lock. The basic concept is to decompose the waterway system into locks (which remain interdependent since they are affected by inflows from adjacent locks), identify the parameters of the interarrival and interdeparture time distributions for each lock, and then estimate the implied waiting times. The structure of the equations used in each module is based as much as possible on queueing theory, while the parameters in those equations are statistically estimated based on simulation results. Currently, the following assumptions are used in the numerical method:

1. Interarrival times and service times are generally distributed.
2. Each lock has one chamber.
3. Inflows and outflows occur only at the two end nodes of a series of locks.
4. The average upstream volumes are equal to the downstream volumes in the long run.
5. The long-run volume to capacity (V/C) ratio is less than 1.0 at every lock.

It should be noted that Assumptions 2, 3, and 4 are only applicable to the numerical method. The simulation model is not limited by those assumptions. The numerical method can provide a quick and inexpensive analysis of lock delays. However, Assumptions 2, 3, and 4 limit fairly significantly the applicability of the currently developed numerical method and necessitate the substitution of the simulation model when significant deviations from those assumptions must be considered. With some extensions to the numerical method, Assumptions 2 and 3 may be eliminated. Assumption 4 could be relaxed fairly easily even though it is usually realistic for waterways. Assumptions 1 and 5 should be kept since they reflect realities rather than analytic limitations.

Structure of Numerical Method

To estimate delays in a queueing system, we need to know the means and variances of the interarrival, interdeparture and service time distributions. For series of G/G/1 queues and bi-directional servers, a difficulty arises in identifying the variances of interarrival and interdeparture times. Because the interarrival times at each lock depend on departures from both upstream and downstream locks, the variances of interarrival times cannot be determined from one-directional scans along a series of queues. To overcome such complex interdependence, an iterative scanning procedure is proposed. The core concept is to decompose the system into individual locks and then sequentially analyze each of those locks. At each lock, the tow arrivals from both directions are first combined into an overall arrival distribution and then split into two directional departure distributions.

The algorithm is initiated by scanning along waterways from either direction, sequentially estimating the interarrival and interdeparture time distributions for each lock. Initially assumed values for the variances of interdeparture times from the opposite direction must be provided for the first scan. Then, the scanning direction is reversed and the process is repeated, using the interdeparture time distributions for the opposite direction estimated in the previous scan. Alternating directions, the scanning process continues until the relative difference in the preselected convergence criteria stays within preset thresholds through successive iterations.

Waiting times at locks can be computed in every iteration (and then used as convergence criteria) or just once after all iterations are completed.

Arrival Processes

The mean and standard deviations of interarrival times are estimated in two steps. First, the means and standard deviations of directional interarrival times at a particular lock are estimated from the interdeparture time distributions of the adjacent locks. If flows are conserved between locks and if the V/C ratio is less than 1, such relations are represented in Eq. 1 (variables are defined in FIGURE 1):

$$\bar{t}_{aji} = \bar{t}_{djk} \quad \begin{cases} k=i-1, & \text{if } j=1 \\ k=i+1, & \text{if } j=2 \end{cases} \tag{1}$$

Because speed variations change headway distributions between locks, Eq. 2 was developed to estimate the standard deviation of directional interarrival times at one lock.

$$\sigma_{aji} = \sigma_{djk} + \underset{(.002)}{.0251}\ln\left(1 + \frac{D_{ik}\sigma_{vik}}{\mu_{vik}}\right) \quad \begin{cases} k=i-1, & \text{if } j=1 \\ k=i+1, & \text{if } j=2 \end{cases} \tag{2}$$

$R^2 = 0.999954 \qquad n = 107 \qquad s_e = 0.0586 \qquad \mu = 5.1685$

This suggests that, theoretically, the standard deviation of directional interarrival times should be equal to the standard deviation of directional interdeparture times plus an adjustment factor depending on the speed distribution and distance.

Second, the overall mean and coefficient of variation of interarrival times for this lock are estimated based on the coefficients of variation of directional interarrival times.

$$\bar{t}_{Ai} = \frac{\bar{t}_{a1i} * \bar{t}_{a2i}}{\bar{t}_{a1i} + \bar{t}_{a2i}} \tag{3}$$

$$\underset{(0.027)\quad(0.014)}{C_{Ai}^{2} = 0.179 + 0.41(C_{a1i}^{2} + C_{a2i}^{2})} \tag{4}$$

$R^2 = 0.9188$ $n = 79$ $s_e = 0.0059$ $\mu = 0.988$

In Eq. 4, the coefficients of variation of upstream and downstream interarrival times carry the same weight in estimating the overall variance of interarrival times, since the mean directional trip rates are equal (Assumption 4).

Departure Processes

The departures module estimates the mean and coefficient of variation of interdeparture times. Based on the flow conservation law, if capacity is not exceeded, the average directional interdeparture equals the corresponding interarrival time:

$$\bar{t}_{dji} = \bar{t}_{aji} \tag{5}$$

The coefficient of variation of interdeparture times is estimated in two steps. First, the coefficient is estimated for combined two-directional departures. Departure processes with generally distributed arrivals and service times are analyzed using Laplace transforms (*8*). Some analytic relations obtained are shown in Dai (*6*). The following metamodel was eventually developed to bypass the difficulties of determining the variances of the lock idle times :

$$C_D^2 = \underset{(0.065)}{0.207} + \underset{(0.066)}{0.795}(C_A^2(1-\rho)+\rho) + \underset{(0.0046)}{1.001}(C_S^2\rho^2-\rho^2) \tag{6}$$

$R^2 = 0.9984 \qquad n = 79 \qquad s_e = 0.0058 \qquad \mu = 0.8311$

Next, the coefficient of variation of directional interdeparture times is estimated. The following metamodel was developed for this purpose:

$$C_{dij}^2 = \underset{(0.0056)}{0.518} + \underset{(0.0068)}{0.491} C_{aij}^2 \, C_{Di}^2 \tag{7}$$

$R^2 = 0.9710 \qquad n = 158 \qquad s_e = 0.013 \qquad \mu = 0.9164$

Delay Function

The delay function is intended to estimate the average waiting time at a lock. By applying Marshall's formula for the variance of interdeparture times (*16*), an exact solution for the average waiting time W, was obtained as follows:

$$W = \frac{\sigma_A^2 + 2\sigma_S^2 - \sigma_D^2}{2\bar{t}_A(1-\rho)} \tag{8}$$

In this delay function, the average waiting time increases as the variance of interarrival and service times increase and decreases as the variance of interdeparture times increases. The average waiting time approaches infinity as the V/C ratio approaches 1.0.

Comparison of Simulated and Numerical Results

To validate the numerical method, its results were compared to the results of the previously validated simulation model. Various system configurations were compared, including the relatively large 20-lock system shown in TABLE 1.

The parameter values for this test system (e.g., means and standard deviations of input distributions and distances between locks) were obtained from random number generators, except for traffic volumes, which were assumed to be 10 tows/day in each direction throughout the system. TABLE 1 shows the input parameters and a comparison of waiting times, which are the output variable of greatest practical interest. It can be seen that the numerical model estimates aggregate waiting times within 7.85% of the simulated ones. At individual locks the percent error can be considerably greater, especially when absolute errors are very small (e.g., in comparisons with zero waiting times). The comparisons of intermediate outputs, e.g., the parameters of directional interarrival and interdeparture time distributions, show that differences below 10% are achieved. The detailed validation results are presented in Dai (*6*).

COMPUTATIONAL TESTS

A number of computational tests have been conducted to investigate the speed, accuracy and convergence properties of the numerical method. Some of the results obtained are presented below. All these results were obtained with the two-directional iterative algorithm (coded in Fortran-77) compiled and executed on an IBM PS/2 model 70 personal computer with an 80386 processor and an 80387 math coprocessor.

Any variable that is computed in every iteration of the algorithm may be used to check for convergence and stop the algorithm when further changes between iterations become arbitrarily small. The most interesting candidate variables for convergence criteria are the variances in the interdeparture times from each lock (which affect error propagation) and the waiting times in queues (which are the output variables of greatest practical economic interest).

The convergence threshold may be specified as a relative change in the value of a variable from one iteration to the next (i.e., a ratio or percentage change) or an absolute difference. The ratios may be very large if and when some variable values approach zero even though absolute differences may be insignificant.

Convergence may be sought based on aggregate or systemwide outputs (e.g., total delay per tow through a series of locks) or may be based on localized outputs (e.g., delay at each lock). In principle, it should be easier to reduce changes between iterations to x percent for a systemwide variable than for every single location in that system.

The original algorithm used the squared coefficients of variation of directional interdeparture times (VARDEP) as the convergence criteria. In this work, the individual lock waiting times (LOCWAIT) and system weighted waiting times (SYSWAIT) are also tested as convergence criteria. Waiting times must then be computed in every iteration rather than just at the end.

The required inputs for the algorithm include the inflow rates, V/C ratio and service time variance at each lock, distances between locks, means and standard deviations of tow speed distributions, and the choice of convergence criterion. We generally used 0.001 as the convergence threshold, i.e., results were considered sufficiently accurate and additional iterations were deemed unnecessary when the variables chosen as convergence criteria changed by less than 0.1% from the previous iteration.

3-Lock Systems

The first test concerns the eight 3-lock systems analyzed in Dai (*6*). These eight systems (described in TABLE 2) were originally used to show the performance of various algorithms. The distances and speed distributions between locks were kept equal within each of these eight systems. Using VARDEP, LOCWAIT, and SYSWAIT as convergence criteria, the estimated individual lock delays and system delays, and number of iterations required are listed in TABLE 3. Also included are the simulated waiting times. Generally, the three criteria perform equally well for each of the eight systems in terms of number of iterations required for convergence. The SYSWAIT criterion produces slightly faster convergence than the others.

While assessing the differences in the number of iterations required with various criteria in System 1, we found that delays at low V/C ratios are so small that relative differences may be large and unstable even for very small changes in the absolute magnitudes of delays. Consequently, more iterations are required to satisfy a relative threshold. If, instead, we set an absolute threshold for delay, e.g., less than 0.001 hr/tow difference between successive iterations, System 1 converges at the fourth iteration for both LOCWAIT and SYSWAIT.

We also sought to check whether the convergence was monotonic, i.e., whether the changes always decrease through successive iterations. We found that relative changes decrease monotonically for all systems when SYSWAIT, but not VARDEP or LOCWAIT are the convergence criteria. However, the magnitudes of various criterion variables change monotonically through successive iterations for all systems, as shown in TABLE 4. It seems that monotonic convergence is more difficult to achieve for local variables when the algorithm scans along the series of locks in alternating directions. When an iteration is defined as a two-way scan (e.g., first upstream, then downstream, and only afterwards compare results to previous iteration) monotonic convergence is achieved for the local variables LOCWAIT and VARDEP.

It is achieved without two-way iterations for the aggregate variable SYSWAIT which, incidentally, requires 3~12% less CPU time than the local criteria.

The algorithm was also allowed to run for 100 iterations to check the convergence and CPU times for various criteria. The results were quite satisfactory since no system ever diverged in this experiment. This is illustrated in FIGURE 2 using System 6 as the example.

20-Lock Systems

To further check the behavior of the algorithm, we randomly generated parameter values for a 20-lock system in which the values of the V/C ratio were uniformly distributed between 0 and 1, and the coefficients of variation of service time were uniformly distributed between 0.2 and 1.0. This test system was assumed to have equal mean inflow rates in the two directions, as well as identical tow speed distributions and distances between any pair of locks. TABLE 5 describes this 20-lock system.

The aggregate results for the 20-lock system are summarized in TABLE 6. We found that the number of iterations required for convergence within 0.001 is almost identical to the numbers in TABLE 3, even though this 20-lock system is more than six times larger. This suggests that the algorithm may be applicable for very large systems. Comparisons of CPU times required for convergence again confirm that the aggregate criterion SYSWAIT saves iterations compared to the local criteria LOCWAIT and VARDEP and reaches convergence with approximately 25% less CPU time. As in 3-lock systems, the 20-lock system never diverges and the monotonic properties with various criteria are similar. With the LOCWAIT criterion a single violation of monotonic convergence was found at lock 2 in the 4th iteration. Consequently, one more scan is desired to bring the entire system into convergence. Such violations were never found when the aggregate convergence criterion SYSWAIT was used or when iterations were defined to consist of two scans in alternate directions.

The relation between system size and computational requirements was also examined using the 20-lock system and arbitrarily chosen subsets of that system. The CPU times and number of iterations required for convergence in various system sizes are shown in FIGURE 3. It again seems very promising that the number of iterations does not change much for different criteria and system sizes. The CPU times seem roughly proportional to system sizes in all cases. FIGURE 3 demonstrates the apparently linear relations. We sought to statistically estimate the relations between CPU time and the number of locks in the system, using the following structural form:

$$CPU_i = K_i \, N^{P_i} \tag{9}$$

In Eq. 9 CPU_i is the central processing time using convergence criterion i, K_i and P_i are statistically estimated parameters associated with criterion i, and N is the number of locks in the system. The P_i parameter was expected to be very close to 1.0, based on the nearly linear relations shown in FIGURE 3, and indeed turned out to be nearly 1.0, confirming the essentially linear relation. The value of P_i was, therefore, fixed at 1.0 and the remaining parameter K_i was estimated as shown in TABLE 7.

The small standard errors and high R^2 again confirm that CPU time is essentially linear with respect to the number of locks in the system. Among the three criteria, the aggregate criterion SYSWAIT has the smallest standard error and highest R^2, suggesting it yields not only the fastest but also the most predictable computer times. The structural form of Eq. 9 forces the computer time function through the origin, since Eq. 9 has no intercept. When an intercept A_i is provided in Eq. 10 (presumably to reflect the fixed times required for setup or input and output functions), even better fits were obtained, as shown in TABLE 7.

$$CPU_i = A_i + K_i N \tag{10}$$

The best fit is again obtained for the SYSWAIT criterion. Thus, based on our very small sample, the best estimate of CPU time (in seconds to reach convergence within 0.001) for N-lock systems is obtained with the SYSWAIT criterion as:

$$CPU = 0.107 + 0.0853N \tag{11}$$

TABLE 6 shows that convergence to within 0.1% difference between successive iterations is reached in 1.75 seconds of CPU time for the 20-lock system and SYSWAIT criterion. The corresponding time for the simulation model to analyze the same 20-lock system on the same computer is 53 minutes per replication, i.e., 1,590 minutes or 95,400 seconds for 30 replications. Thus, in this case simulation requires 54,514 times more CPU time than the numerical method. However, it should be noted that our simulation runs were designed to extract very precise estimates for estimating new metamodels. We usually simulated 22,000 tows, discarded the first 10,000 of those, and replicated the simulation 30 to 80 times for each "data point". For practical application, the simulation would require 10^4 to 10^5 times more CPU time than the numerical method.

Double Scanning Versus Single Scanning

In the baseline algorithm an iteration consists of scanning the waterway from one end to the other, i.e., in one direction. The next iteration would then scan in the opposite direction. The results obtained so far suggest that a smoother convergence may be obtained by double scanning, i.e., checking for convergence only after two full scans in opposite directions are completed. With such double scanning, the changes in variables are always found to decrease (or at least not increase) with each successive convergence check, which is performed every second iteration by comparing iteration i with iteration i-2 (instead of i-1).

However, double scanning imposes a computer time penalty by increasing the number of iterations required for convergence to a specified threshold. That is shown in TABLE 8 where the convergence threshold is still 0.001. There are two reasons for the penalty. First, an even number of iterations is required in double scanning, even when convergence is reachable with one less iteration. Second, a larger change may be expected after two iterations than after one, making the same threshold (e.g., 0.001) harder to satisfy.

Thus, it seems that double scanning provides added reassurance that the algorithm converges in a smooth and well behaved way. However, since convergence seems so assured regardless of scanning procedure, it seems preferable to opt for the computation savings of single scanning.

CONCLUSIONS

A numerical method has been developed to estimate waterway travel times through a series of lock queues. This numerical method was estimated from simulation results. It can approximately duplicate simulation results for complex systems of interdependent queues, while requiring

$10^4 \sim 10^5$ times less computer time than simulation. The basic approach used in this numerical method and several of its components (or "metamodels") should lead to numerical analysis methods for other types of queueing networks with greater complexity.

This paper focused on the main computational characteristics of the baseline numerical method and some of its variations. The main computational findings are as follows:

1. Variables other than the original interdeparture time variance VARDEP are suitable as convergence criteria. In particular, the aggregate waiting time SYSWAIT yields convergence faster than the other variables considered. Not surprisingly, more iterations may be needed if a specified convergence threshold (e.g., 0.1%) is to be satisfied at every location and in every direction rather than for an aggregate criterion.
2. Convergence to within 0.1% of values in the previous iteration is achieved relatively quickly (typically in 4~6 iterations), even when that 0.1% threshold must be satisfied everywhere in a 20-lock system.
3. Convergence is achieved smoothly and, with rare exceptions, differences in the variable values decrease with each successive iteration. The exceptions are all traceable to scans in alternating directions, and can be avoided by double scanning before convergence checks or by always scanning in the same directions. However, since convergence seems always assured, the single scanning in alternating directions seems preferable to save computer time.
4. The computer time required by the algorithm seems to be linear with respect to the number of locks in the system. It also seems to be quite predictable. Thus, the numerical method should analyze efficiently relatively large systems of interdependent queues.

ACKNOWLEDGEMENT

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TABLE 1 VALIDATION OF NUMERICAL METHOD FOR 20-LOCK TEST CASE

Lock	σ_A[a]	C_A[b]	σ_D	C_D	σ_S	C_S	V/C	Dist[c]
1	1.21	1.01	0.92	0.77	0.52	0.56	0.78	7.04
2	1.18	0.98	1.17	0.98	0.10	0.70	0.12	49.04
3	1.20	1.00	0.91	0.76	0.74	0.69	0.90	46.05
4	1.19	0.99	1.05	0.88	0.69	0.76	0.75	47.74
5	1.20	1.00	1.02	0.85	0.81	0.78	0.86	105.56
6	1.19	0.99	0.90	0.75	0.61	0.60	0.84	71.76
7	1.19	0.99	1.05	0.88	0.91	0.84	0.90	39.91
8	1.21	1.01	1.20	1.00	0.13	0.57	0.19	91.12
9	1.19	0.99	0.94	0.79	0.65	0.65	0.83	60.55
10	1.17	0.97	0.99	0.83	0.75	0.74	0.85	22.44
11	1.21	1.01	1.08	0.90	0.56	0.71	0.66	53.38
12	1.22	1.02	1.20	1.00	0.23	0.67	0.28	89.78
13	1.22	1.02	1.19	0.99	0.28	0.69	0.34	103.77
14	1.23	1.02	0.89	0.74	0.57	0.57	0.83	125.02
15	1.21	1.01	1.16	0.97	0.35	0.71	0.41	105.41
16	1.22	1.02	1.18	0.99	0.37	0.80	0.39	80.29
17	1.21	1.01	1.02	0.85	0.45	0.57	0.66	99.98
18	1.20	1.00	0.95	0.79	0.62	0.64	0.81	65.54
19	1.18	0.99	1.13	0.94	0.45	0.74	0.51	2.38
20	1.22	1.02	0.96	0.80	0.71	0.70	0.85	96.75

Lock	Estimated Waiting Time, hrs/tow			
	Numerical	Simulation	Difference	%
1	2.15	2.04	0.11	5.31
2	0.00	0.01	-0.01	--[d]
3	6.91	6.37	0.54	8.48
4	1.91	1.78	0.12	6.88
5	4.71	4.20	0.50	11.96
6	3.39	2.67	0.73	27.19
7	7.76	7.23	0.53	7.38
8	0.00	0.04	-0.03	--
9	3.30	2.83	0.47	16.48
10	4.21	3.73	0.49	13.02
11	1.10	1.08	0.01	1.04
12	0.08	0.10	-0.03	-27.02
13	0.13	0.17	-0.04	-22.84
14	3.33	3.20	0.13	4.19
15	0.22	0.26	-0.04	-16.41
16	0.21	0.26	-0.05	-19.91
17	0.95	0.99	-0.04	-4.26
18	2.80	2.74	0.06	2.29
19	0.42	0.45	-0.03	-7.54
20	4.34	4.27	0.08	1.81
System	47.92	44.44	3.49	7.85

[a]σ_i: Standard deviation of interarrival time, interdeparture time, and service time distributions, respectively.
[b]C_i: Coefficients of variation of interarrival time, interdeparture time, and service time distributions, respectively.
[c]Dist: Distance to the next lock, in miles.
[d]Not applicable.

TABLE 2 PHYSICAL CHARACTERISTICS OF 3-LOCK SYSTEMS

System	Lock	Two-way Flow Rate tows/day	V/C	Distance miles	Tow Speed miles/day μ_v[a]	σ_v[b]	Variance of Service Time hr^2/tow^2
1	1	6.0	0.01	5	270	85	0.0007
	2	6.0	0.07	5	270	85	0.0360
	3	6.0	0.17	5	270	85	0.1897
2	1	12.0	0.15	5	325	102	0.0309
	2	12.0	0.34	5	325	102	0.1620
	3	12.0	0.25	5	325	102	0.0915
3	1	18.0	0.22	5	108	34	0.0309
	2	18.0	0.03	5	108	34	0.0006
	3	18.0	0.50	5	108	34	0.1618
4	1	24.0	0.50	5	162	51	0.1883
	2	24.0	0.29	5	162	51	0.0646
	3	24.0	0.67	5	162	51	0.3330
5	1	27.0	0.75	10	108	34	0.2271
	2	27.0	0.57	10	108	34	0.1279
	3	27.0	0.89	10	108	34	0.3167
6	1	27.0	0.75	20	216	68	0.1616
	2	27.0	0.57	20	216	68	0.0909
	3	27.0	0.89	20	216	68	0.2259
7	1	28.5	0.60	5	325	102	0.1557
	2	28.5	0.05	5	325	102	0.0011
	3	28.5	0.80	5	325	102	0.2738
8	1	28.5	0.35	60	162	51	0.0645
	2	28.5	0.60	60	162	51	0.1882
	3	28.5	0.80	60	162	51	0.3332

[a]μ_v: Average tow speed.
[b]σ_v: Standard deviation of tow speeds.

TABLE 3 COMPUTATIONAL COMPARISON FOR VARIOUS CRITERIA IN 3-LOCK SYSTEMS

		Estimated Waiting Time, hrs/tow						
			VARDEP		LOCWAIT		SYSWAIT	
System	Lock	Wsim[a]	Wv[b]	Dv[c]	W_L	D_L	Ws	Ds
1	1	0.0003	0.0001	-0.0002	0.0001	-0.0002	0.0001	-0.0002
	2	0.0153	0.0175	0.0022	0.0176	0.0023	0.0176	0.0023
	3	0.0989	0.0990	0.0001	0.0990	0.0001	0.0990	0.0001
	Total	0.1145	0.1166	0.0021	0.1167	0.0022	0.1167	0.0022
Required Iterations			5		7		6	
2	1	0.0334	0.0290	-0.0044	0.0290	-0.0044	0.0290	-0.0044
	2	0.2316	0.2289	-0.0027	0.2289	-0.0027	0.2289	-0.0027
	3	0.1139	0.1099	-0.0040	0.1099	-0.0040	0.1099	-0.0040
	Total	0.3789	0.3678	-0.0111	0.3678	-0.0111	0.3678	-0.0111
Required Iterations			5		5		5	
3	1	0.0542	0.0528	-0.0014	0.0528	-0.0014	0.0528	-0.0014
	2	0.0008	0.0001	-0.0007	0.0001	-0.0007	0.0001	-0.0007
	3	0.4621	0.4660	0.0039	0.4660	0.0039	0.4659	0.0038
	Total	0.5171	0.5189	[illegible]018	0.5189	0.0018	0.5188	0.0017
Required Iterations			5		5		4	
4	1	0.4355	0.4404	0.0049	0.4404	0.0049	0.4404	0.0049
	2	0.0962	0.0999	0.0037	0.0999	0.0037	0.0999	0.0037
	3	1.2028	1.1844	-0.0184	1.1844	-0.0184	1.1844	-0.0184
	Total	1.7345	1.7247	-0.0098	1.7247	-0.0098	1.7247	-0.0098
Required Iterations			4		4		4	
5	1	1.3926	1.4693	0.0767	1.4693	0.0767	1.4693	0.0767
	2	0.3901	0.4127	0.0226	0.4127	0.0226	0.4127	0.0226
	3	4.9837	4.7980	-0.1857	4.7980	-0.1857	4.7980	-0.1857
	Total	6.7664	6.6800	-0.0864	6.6800	-0.0864	6.6800	-0.0864
Required Iterations			4		4		4	
6	1	1.2203	1.3038	0.0835	1.3038	0.0835	1.3038	0.0835
	2	0.3286	0.3416	0.0130	0.3416	0.0130	0.3416	0.0130
	3	4.4608	4.2983	-0.1625	4.2983	-0.1625	4.2983	-0.1625
	Total	6.0097	5.9437	-0.0660	5.9437	-0.0660	5.9437	-0.0660
Required Iterations			4		4		4	
7	1	0.5430	0.5900	0.0470	0.5899	0.0469	0.5899	0.0469
	2	0.0012	0.0001	-0.0011	0.0001	-0.0011	0.0001	-0.0011
	3	2.0874	2.0906	0.0032	2.0906	0.0032	2.0906	0.0032
	Total	2.6316	2.6807	0.0491	2.6806	0.0490	2.6806	0.0490
Required Iterations			4		3		3	
8	1	0.1372	0.1405	0.0033	0.1405	0.0033	0.1405	0.0033
	2	0.6381	0.6592	0.0211	0.6592	0.0211	0.6592	0.0211
	3	2.3165	2.3146	-0.0019	2.3146	-0.0019	2.3146	-0.0019
	Total	3.0918	3.1143	0.0225	3.1143	0.0225	3.1143	0.0225
Required Iterations			4		4		4	

[a]Wsim: Waiting time estimated from simulation.
[b]Wi: Waiting time estimated when criterion i used.
[c]Di: Difference between numerically estimated waiting time at a given iteration and simulated waiting time = Wi - Wsim.

TABLE 4 CONVERGENCE PROPERTIES FOR VARIOUS CRITERIA IN SYSTEM 2

Criterion: VARDEP

	Magnitude, Dir 1			Relative Difference, Dir 1		
Iter	Lock 1	Lock 2	Lock 3	Lock 1	Lock 2	Lock 3
1	0.9930	0.9629	0.9715	--[a]	--	--
2	1.0027	0.9724	0.9715	0.0098	0.0098	0.0000
3	1.0027	0.9778	0.9802	0.0000	0.0056	0.0090
4	1.0030	0.9780	0.9802	0.0003	0.0002	0.0000
5	1.0030	0.9782	0.9804	0.0000	0.0002	0.0002

	Magnitude, Dir 2			Relative Difference, Dir 2		
Iter	Lock 1	Lock 2	Lock 3	Lock 1	Lock 2	Lock 3
1	0.9456	0.9214	0.9889	--	--	--
2	0.9884	0.9705	0.9889	0.0453	0.0533	0.0000
3	0.9884	0.9715	0.9907	0.0000	0.0011	0.0018
4	0.9897	0.9725	0.9907	0.0013	0.0010	0.0000
5	0.9897	0.9726	0.9907	0.0000	0.0000	0.0000

Criterion: LOCWAIT

	Magnitude			Relative Difference		
Iter	Lock 1	Lock 2	Lock 3	Lock 1	Lock 2	Lock 3
1	0.0177	0.1994	0.1065	--	--	--
2	0.0287	0.2254	0.1065	0.6167	0.1304	0.0000
3	0.0287	0.2283	0.1098	0.0000	0.0126	0.0312
4	0.0290	0.2288	0.1098	0.0110	0.0023	0.0000
5	0.0290	0.2289	0.1099	0.0000	0.0004	0.0007

Criterion: SYSWAIT

	Magnitude	Relative Difference
Iter	System	System
1	0.3236	--
2	0.3606	0.1142
3	0.3667	0.0171
4	0.3676	0.0023
5	0.3677	0.0004

[a]Not applicable.

TABLE 5 RELEVANT DATA FOR THE 20-LOCK SYSTEM

Lock	V/C	C_s[a]	Cap[b]	μ_s[c]	σ_s^2[d]
1	0.5625	0.2482	48	0.5000	0.0154
2	0.2473	0.2591	109	0.2198	0.0032
3	0.4505	0.3725	60	0.4004	0.0223
4	0.4098	0.2942	66	0.3643	0.0115
5	0.9865	0.8953	27	0.8769	0.6163
6	0.2148	0.2328	126	0.1909	0.0020
7	0.8315	0.5422	32	0.7391	0.1606
8	0.7088	0.3447	38	0.6300	0.0472
9	0.8563	0.9832	32	0.7612	0.5601
10	0.5989	0.8641	45	0.5324	0.2116
11	0.2065	0.6823	131	0.1836	0.0157
12	0.0510	0.5392	529	0.0453	0.0006
13	0.9894	0.8309	27	0.8795	0.5340
14	0.5051	0.4834	53	0.4490	0.0471
15	0.6715	0.6363	40	0.5969	0.1442
16	0.6728	0.7805	40	0.5980	0.2179
17	0.9475	0.6943	28	0.8422	0.3419
18	0.8662	0.4078	31	0.7700	0.0986
19	0.9074	0.4017	30	0.8066	0.1050
20	0.8711	0.9968	31	0.7743	0.5957

Inflow Rate of Direction 1 (tows/day)	13.5
Inflow Rate of Direction 2 (tows/day)	13.5
Convergence Threshold	0.001
Tow Speed (miles/day)	213.48
Standard Deviation of Speed (miles/day)	67.68
Distance between Locks (miles)	20.0

[a]C_s: Coefficient of variation of service time distribution.

[b]Cap: Lock capacity, tows/day.

[c]μ_s: Mean of service time distribution, hrs/tow.

[d]σ_s^2: Variance of service time distribution, hrs^2/tow^2.

TABLE 6 COMPUTATION RESULTS FOR THE 20-LOCK SYSTEM

	VARDEP	LOCWAIT	SYSWAIT
Required Iterations for Convergence Within 0.001	4	5	4
CPU Time (seconds)	2.15	2.36	1.75
Total Waiting Time (hrs/tow)	151.2056	151.2044	151.2056
100 Iterations			
Divergence	None	None	None
CPU Time (seconds)	31.25	34.38	27.14
Total waiting time (hrs/tow)	151.2043	151.2043	151.2043

TABLE 7 PARAMETERS FOR CPU TIME VS. SYSTEM SIZE

Criterion	K_1	Standard Error of K_1	Standard Error of CPU Estimate	R^2	A_1
Eq. 9					
VARDEP	0.1129	0.0026	0.0774	0.9867	
LOCWAIT	0.1245	0.005	0.1482	0.9537	
SYSWAIT	0.0925	0.0019	0.0578	0.9885	
Eq. 10					
VARDEP	0.106	0.0055	0.0696	0.9919	0.102
LOCWAIT	0.108	0.0084	0.1074	0.9817	0.242
SYSWAIT	0.0853	0.0024	0.0315	0.9974	0.107

TABLE 8 ITERATIONS REQUIRED FOR VARIOUS SCANNING PROCESSES

3-Lock System 1	VARDEP	LOCWAIT	SYSWAIT
Single Scan	5	7	6
Double Scan	6	8	8
20-Lock System			
Single Scan	4	5	4
Double Scan	6	6	6

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FIGURE 1 STRUCTURE OF NUMERICAL METHOD

FIGURE 2 CONVERGENCE FOR 3-LOCK SYSTEM WITH VARIOUS CRITERIA

FIGURE 3 RELATIONS BETWEEN SYSTEM SIZE AND COMPUTATION SPEED

Arrival Process

$$\bar{t}_{aji} = \bar{t}_{djk} \quad \begin{cases} k=i-1, & \text{if } j=1 \\ k=i+1, & \text{if } j=2 \end{cases} \qquad (1)$$

$$\sigma_{aji} = \sigma_{djk} + .0251\ln\left(1+\frac{D_{ik}\sigma_{vik}}{\mu_{vik}}\right) \quad \begin{cases} k=i-1, & \text{if } j=1 \\ k=i+1, & \text{if } j=2 \end{cases} \qquad (2)$$

$$\bar{t}_{Ai} = \frac{\bar{t}_{a1i} * \bar{t}_{a2i}}{\bar{t}_{a1i} + \bar{t}_{a2i}} \qquad (3)$$

$$C_{Ai}^2 = 0.179+0.41(C_{a1i}^2 + C_{a2i}^2) \qquad (4)$$

Departure Process

$$\bar{t}_{dji} = \bar{t}_{aji} \qquad (5)$$

$$C_D^2 = .207+.795(C_A^2(1-\rho)+\rho)+1.001(C_S^2\rho^2-\rho^2) \qquad (6)$$

$$C_{dji}^2 = 0.518+0.491C_{aji}^2\,C_{Di}^2 \qquad (7)$$

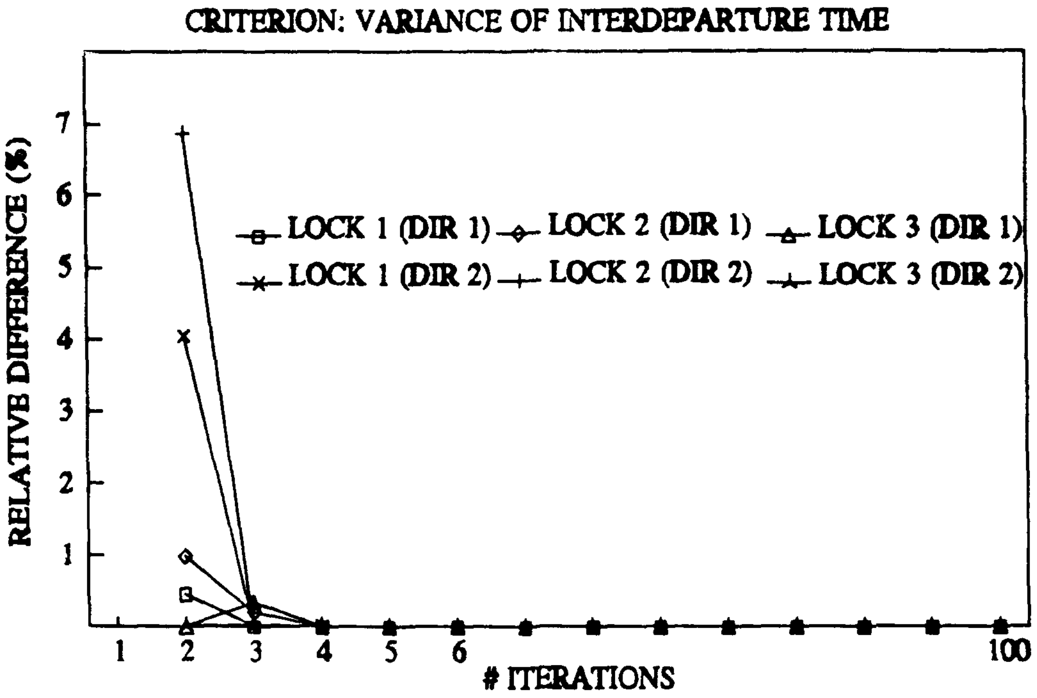
Delay Function

$$W = \frac{\sigma_A^2+2\sigma_S^2-\sigma_D^2}{2\bar{t}_A(1-\rho)} \qquad (8)$$

Notation:

C_{Ai} : coefficient of variation of interarrival times at Lock i
C_{aji} : coefficient of variation of directional interarrival times for Direction j and Lock i
C_{Di} : coefficient of variation of interdeparture times for Direction j and Lock i
C_{dji} : coefficient of variation of directional interdeparture times for Direction j and Lock i
C_S : coefficient of variation of service times
D_{ik} : distance between Locks i and k
i : index of currently scanned lock
j : direction index (1 = downstream, 2 = upstream)
k : index of adjacent locks
$\bar{t}_{Ai}$: mean interarrival time at Lock i
$\bar{t}_{aji}$: mean interarrival time for Direction j and Lock i
$\bar{t}_{djk}$: mean interdeparture time for Direction j and Lock k
μ_{vik} : mean tow speed between Locks i and k
σ_{aji} : standard deviation of interarrival times for Direction j and Lock i
σ_{djk} : standard deviation of interdeparture times for Direction j and Lock k
σ_{vik} : standard deviation of tow speeds between Locks i and k

FIGURE 1 STRUCTURE OF NUMERICAL METHOD



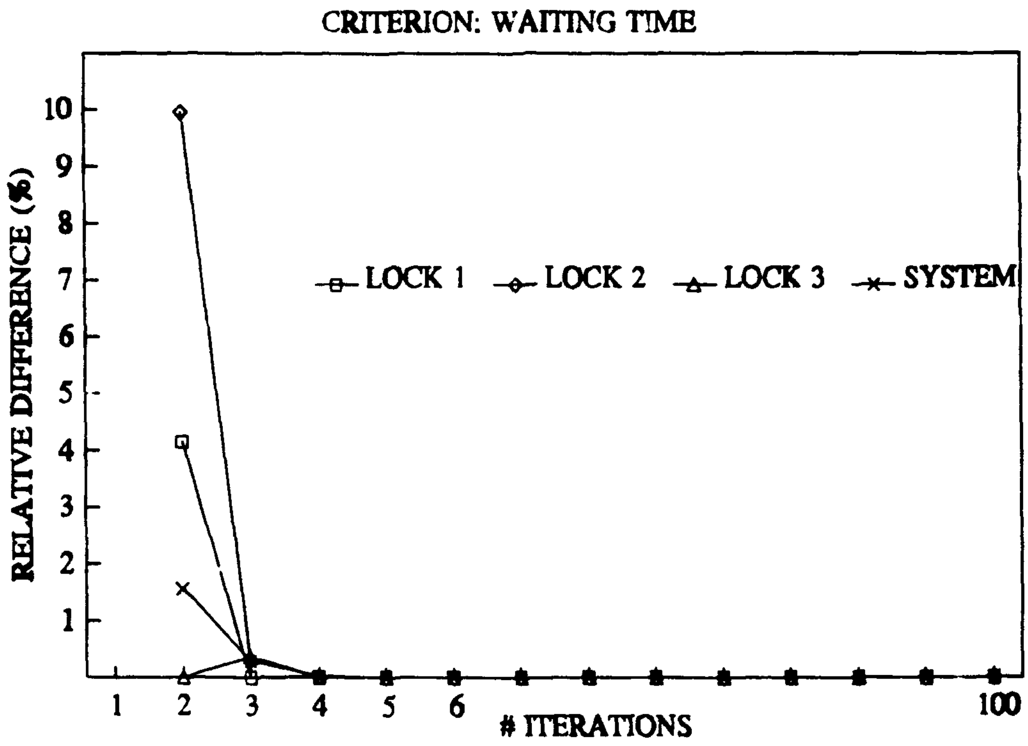


FIGURE 2 CONVERGENCE FOR 3-LOCK SYSTEM WITH VARIOUS CRITERIA

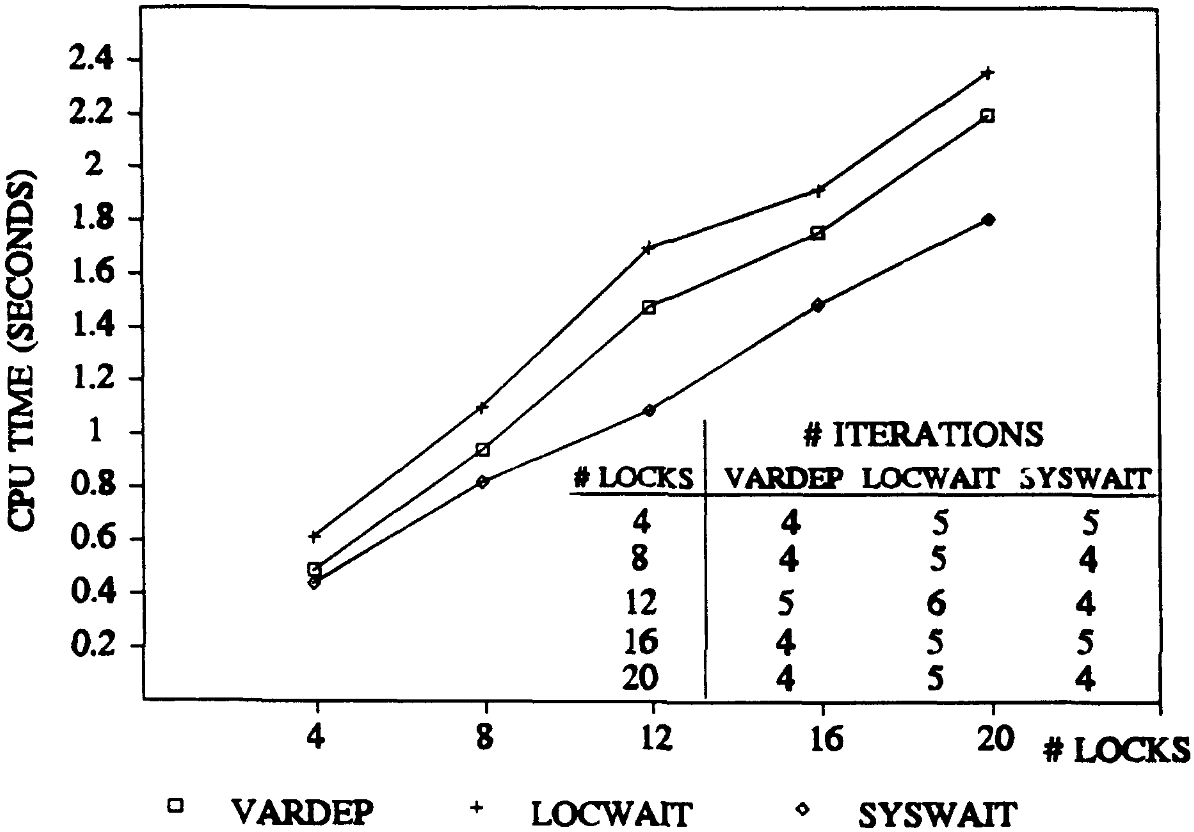


FIGURE 3 RELATIONS BETWEEN SYSTEM SIZE AND COMPUTATION SPEED

A METHODOLOGY FOR PLANNING EFFICIENT INVESTMENTS ON INLAND WATERWAYS

by

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A Methodology for Planning Efficient Investments on Inland Waterways

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Abstract

This paper presents a methodology that addresses the analytic complications associated with making investment planning decisions for inland waterway improvements. These complicatons include interdependencies between locks, bidirectional traffic, stalls, dual chamber facilities, and budget limitations. The methodology address most steps of the investment planning process for locks, namely project evaluation, sequencing, and scheduling.

1. Introduction

The national waterway study, and other navigation studies identified a need for substantial investment in the waterway infrastructure derived from several trends and observations. The first trend is that lock conditions are deteriorating, giving rise to an increase in tow delays. Currently there are about 100 locks that have exceeded their design life. The second is that traffic levels are consistently increasing for many locks in the system. Also, prospects for increased grain exports are improving. Currency reform, the grain export enhancement program, reduction in worldwide carryover stocks of grain, and other factors have contributed to increases in exports. A third trend is an increase in tow sizes. While this tends to increase overall transport efficiency, large tows must be disassembled into several pieces to move through the chamber and must later be reassembled. The fourth observation is that additional funding sources for major lock rehabilitation projects is not likely. The major sources of funding for such projects are the Federal matching share and fuel tax receipts. The Federal share of 50 percent and the fuel tax rate of 20 cents beyond 1995 are not likely to increase in the near future.

The trends identified by these studies present interesting but challenging opportunities for developing a more comprehensive methodology for inland waterway planning and operations analysis. The following are the primary analytical needs in developing such a methodology:

1. more reliable forecasting methods,
2. more reliable techniques for predicting delays at locks,
3. identification and assessment of the benefits of lock rehabilitation, and
4. more efficient techniques for sequencing and scheduling lock improvement projects.

This paper is an overview of a methodology designed to address many of the analytical needs resulting from trends in conditions, traffic levels, and funding sources for waterway locks. Particular emphasis is placed on satisfying items 2 and 4 above. It is the product of several research projects conducted over the last four years through the Institute for Water Resources and consists of the following components:

1. exploratory data analysis and characterization of problems,
2. a microsimulation model of waterway traffic and lock operations,
3. statistically estimated functions ("metamodels") to approximate the results of the simulation model,
4. an algorithm for prioritizing and scheduling proposed lock improvement projects, and
5. a computer program for cash flow analysis for the Inland Waterway Trust Fund.

2. Background

There are numerous analysis tools available to assist in modeling lock operations and investment parameters. These include benefit-cost analysis, mathematical programming, queuing theory, and simulation. There exist some significant works on the application of these tools to waterway problems. However extensions to the previously available methods were necessary to meet the analytical demands of current U.S. waterway transportation problems.

2.1 Determining Delays at Locks (Analytic Models)

Two single-lock models based on the application of queuing theory have been found for estimating lock delays. DeSalvo and Lave [6] represent the lock operation as a simple server queuing process with Poisson distributed arrivals and exponentially distributed service times. However, these assumed distributions do not adequately fit the physical system of locks on waterways [5]. Wilson [15] improved on this model by treating the service processes as general distributions rather than exponentially distributed, which is far more realistic [5]. However, this was for single chamber locks only and the Poisson arrivals assumption is not realistic for all locks. Two other deficiencies exist in both of the above models. First, neither of these models accounts for stalls. Stalls cause service interruptions at locks, thus reducing lock capacities or increasing delays. Their occurrence is very difficult to predict. Secondly, both models were developed to analyze delays at a single isolated lock. Since the delays at adjacent locks may be highly interdependent, it is desirable to analyze lock delays for entire systems.

Queuing theory offers some solutions for more general queuing systems, i.e. those beyond Poisson arrivals and exponential service times (M/M/1). In special cases combining Poisson arrivals with general service times (M/G/1) and general arrivals with exponential service times (G/M/1), closed-form solutions for the mean waiting time have been obtained [15]. G/G/1 queues are difficult cases in queuing theory and the available techniques for handling them are incomplete. Solving G/G/M queues is even more difficult than solving G/G/1 queues. The methods of approximations and bounds have been proposed to solve G/G/M queues [11,12]. These can be accurate and efficient under heavy traffic conditions. However, the methods are difficult to extend to the a series and networks of queues found in waterways.

2.2 Determining Delays at Locks (Simulation Models)

An early microscopic simulation models to analyze lock delays and tow travel times was developed by Howe [7]. In that model, service times were based on empirically-determined frequency distributions. To avoid some troublesome problems and errors associated with the requirement to balance long-run flows in Howe's model, Carrol and Bronzini [4] developed another simulation model.

It provided detailed outputs on such variables as tow traffic volumes, delays, processing times, transit times, average and standard deviations of delay and transit times, queue lengths, and lock utilization ratios. Each of these models simulates waterway operations in detail, but requires considerable amounts of data and computer time, which limit their applicability for problems with large networks and numerous combinations of improvement alternatives. They both assume Poisson distributions for tow-trip generation, which is not always realistic. Moreover, service failures ("stalls") which are very different in frequency and duration from other events and have significant effects on overall transit-time reliability, are not accounted for. Hence a waterway simulation model that explicitly accounts for stalls is desirable for evaluating and scheduling lock improvement projects.

2.3 Benefit-Cost Analysis for Interdependent Improvement Projects

The delays at locks have been shown to be interdependent, i.e. the delays at one lock are related to the delays at one or more other locks [10]. That is because the departure process from one queuing station (e.g. a lock) in a network effects the arrival process at the next queues in that network. Interdependence not only yields difficulties in predicting lock delays, but also in conducting benefit-cost analysis. Current methods of benefit-cost analysis are quite satisfactory for analyzing mutually exclusive projects, and reasonably satisfactory for independent projects.

However, there is a void in analyzing projects that are interdependent.

In evaluating and sequencing mutually exclusive projects, the net present value and benefit-cost ratio methods as discussed in [2] can be used if the benefits and costs are quantifiable and can be accurately assessed over the planning horizon. This is because, in such cases, the benefits and costs of projects are not dependent on the project set selected. When working with independent projects, we can use an integer programming approach where the objective is to maximize the sum of net present values subject to a set of budget constraints [8].

However, for interdependent projects, the estimates of benefits and costs must be performed simultaneously with project selection. Therefore it may be necessary to enumerate all possible project *combinations* when *selecting* a set of projects and all possible *permutations* when *sequencing* a set of projects. However, as a practical matter, complete enumeration becomes infeasible as a method of finding optimal combinations and permutations of projects as the number of projects becomes even modestly large. An alternative to complete enumeration is an augmented integer programming formulation. Such methods are discussed in [8]. To capture some of the interdependence, the objective function includes "interaction terms" for pairs of projects. These terms represent the deviation from linear addition when summing the net present values for two interdependent projects. For example, if two Projects *A* and *B* are independent the net present values may be summed linearly:

$$NPV_{AB} = NPV_A + NPV_B. \tag{1}$$

Alternatively, for interdependent projects an interaction term is added:

$$NPV_{AB} = NPV_A + NPV_B + d_{AB}. \tag{2}$$

There are significant shortcomings with such an approach. First, only paired interactions are represented; depending on the application, three, four, or more projects may be simultaneously dependent. Second, the number of integer variables and interaction terms is excessive. The estimation of interaction terms is quite complex for most applications. While many problems may be smaller than this example, most integer programming algorithms have serious difficulties with problems of this size.

There is a need to formulate the selection and sequencing of interdependent projects in a manner that is not computational intractable and does not require excessive estimation of interaction terms. It seems that overcoming these voids requires the development of a method whereby the numerous permutations of possible programs may be efficiently represented and searched (without complete enumeration) and the determination of efficient project implementation schedules.

3. Components of the Methodology

3.1 Simulation Model

In light of the many shortcomings and difficulties associated with analytic methods of estimating delays at locks, a simulation model has been developed to analyze tow operations along waterways. The model may be used to determine the relations among delays, tow trips, distributions of generated travel times, and coal consumption and inventories. The model can account for the stochastic effects and seasonal variations and can estimate the following: tow delays at each lock, interarrival and interdeparture-time distributions for each lock and for each direction, tow travel times along the waterway, inventory levels and expected stock-out amounts for commodities delivered by waterway, and many other variables of interest to waterway users and operators. Development of the model was based on the Lock Performance Monitoring System (LPMS) data. The model is event scanning, with four types of events initiating a status update: 1) stochastic generation of tow trips; 2) tow entrances at locks as determined by arrival times, chamber availability, and chamber assignment discipline; 3) the arrival of a tow at its destination; and 4) the occurrence of stalls at a chamber.

There are several features of this model that lend itself well to waterway operations. The simulation model is microscopic, i.e. it traces the movement of each individual to and records its characteristics, including the number of barges, commodity types, speed, origin and destination, travel direction, and arrival time at various points. Any distributions for trip generation, travel speeds, lock service times and tow size may be handled by the model. These distributions can be specified for each interval in tables or by standard statistical distributions. Tows are allowed to overtake other tows and the model simulates two-way traffic through common servers and accounts for stalls. The size of waterway systems that the model can handle is limited only by computer and compiler capacity. Further, the model has been developed with "dynamic dimensioning," for additional increases in flexibility in modeling various waterway systems.

The main simplifying assumptions in the current version of the simulation model are as follows:

1. The tows maintain a constant size through the entire trip.
2. The service discipline is First-In-First-Out as are operations on the Mississippi and Ohio Rivers.
3. The queue storage area is unlimited.
4. The tow speeds are normally distributed and constant for each round trip.
5. The time intervals between two successive stalls and the stall durations are exponentially distributed.

These assumptions are not seriously restrictive, but can still be easily modified.

The simulation model consists of five operation routines and one scheduler routine. The operation routines are associated with the five types of events and are invoked by the scheduler. Figure 1 is a chart of the flow of data as dictated by the model.

To check the logic of this simulation model, its results were first compared to theoretical (but very well established) results from queuing theory. The results of the model were then compared with observed data to demonstrate how closely the model represents real systems and verify its ability to simulate the special features of waterways.

A *partial* validation of the model is possible by comparing the model to theoretical results for the *special case* of Poisson arrivals and generally distributed service times. The waiting times predicted by the simulation model at a single lock were compared with those obtained from queuing theory for this special case. A validation has been conducted for a variety of volume/capacity (V/C) ratios ranging from 0.0471 to 0.8934. To reduce the variance of the output each result was obtained by averaging the output from 30 independent simulation runs. To insure results were compared for a steady state, each simulation run discarded the first 10,000 tow waiting times and collected the next 12,000 values for computing the average waiting time.

INPUT:

Link & Lock Characteristics
Traffic Demand
Probability Distributions
Inventories & Consumption

PROCESS:

Origin Nodes: generating tow trips
Destination Nodes: updating: cumulative deliveries
cumulative consumption
inventory levels
Locks: assigning chamber
determining number of cuts
determining lock service times
calculating queuing times
Links: determining traveling times
determining arrival times to next locks or destinations

OUTPUT:

Average waiting time per tow at each lock
Average waiting time per tow at each lock for each O-D pair
Means and variances of interarrival and interdeparture times at each lock
Cumulative deliveries, cumulative consumption, inventory levels
Average speed
Total number of tow trips for different O-D pairs
Total queuing times for different locks
Total lock service times for different locks and chambers
Total tow travel times & distances

Figure 1 Structure and Elements of The Simulation Model

The results, shown in Table 1, confirm that the simulated and theoretical average waiting times are extremely close. Such results verify that the overall mechanism of the simulation model is correct. They also show that generally distributed service times are generated satisfactorily in the simulation model. That is reassuring since the same logic is also used to generate generally distributed interarrival times for G/G/1 queues and, ultimately to develop metamodels for series of G/G/1 queues.

Table 1. Comparison of Theoretical and Simulated Results for a Single Lock Queue (M/G/1)

V/C	T_A *1 Avg (hr)	T_A *1 Var (hr^2)	T_S *2 Avg (hr)	T_S *2 Var (hr^2)	W_{sim} *3 (hr)	W_t *4 (hr)	Devia. *5 (%)
0.893	0.888	0.789	0.793	0.319	4.9516	5.0059	-1.09
0.755	0.888	0.789	0.670	0.227	1.5575	1.5522	0.34
0.566	0.888	0.789	0.503	0.128	0.4926	0.4935	-0.19
0.330	0.888	0.789	0.293	0.044	0.1082	0.1087	-0.46
0.047	0.888	0.789	0.042	0.001	0.00155	0.00156	-0.64

*1 T_A: interarrival times
*2 T_S: service times
*3 W_{sim}: average waiting times from simulation
*4 W_t: average waiting times from queuing theory
*5 Devia.: deviation which is defined as $(W_{sim}-W_t)/W_t*100\%$

The simulation results were then compared with the observed data at Locks 22, 24, 25, 26, and 27 on the Mississippi River. These locks were selected based on their criticality and available data. The five locks were simulated as an interacting series. Some of the validation results are summarized in Table 2. Each result is averaged from 80 independent simulation runs. Table 2 shows that the difference between the simulated and observed average waiting times for each lock are within the 95% confidence interval based on the t test, except at Lock 25. The observed data also show that tows sometimes were kept waiting at Lock 25 even when the chamber was idle. Therefore, no direct comparison of average waiting times at Lock 25 is appropriate.

Table 2. Comparison of Simulated and Observed Average Waiting Times

Lock	W_{sim} *1 (min)	W_{obs} *2 (min)	Difference (min)	95% Confidence Interval
22	4.09	3.73	0.36	3.49
24	6.12	6.36	0.24	6.72
25	4.49	10.94	6.45	- *3
26	119.40	130.99	11.59	60.73
27	36.49	34.43	2.06	23.92

*1 W_{sim}: simulated average waiting times
*2 W_{obs}: observed average waiting times
*3 The comparison is not appropriate.

3.2 Metamodel Approximations to Simulation

Each simulation run takes from a few seconds to a few minutes on a personal computer depending on the values of various problem parameters. Despite this high level of efficiency, simulation time becomes expensive for evaluating large combinatorial problems such as investment planning. Furthermore, the project combinations may have to be evaluated over several time periods. A metamodeling approach which statistically estimates unknown parameters of equations from simulation results and then uses these equations as substitutes has been developed to overcome the computational requirements of simulation. The main difficulty with this approach is to find structural forms for the approximating functions which fit the simulation results as well as possible, This was accomplished by queuing theory insofar as possible for these functions.

In this study, a numerical method has been developed for estimating delays through a series of queues. The method decomposes systems of queues into individual queuing stations. The analysis of each queuing station is further decomposed into three modules, namely arrival processes, departure processes, and delay functions. Arrival processes at a particular lock depend on the departure distributions from the upstream and downstream locks and the intervening speed distributions. The departure processes depend on the interaction among the arrival distributions and service time distributions at one lock. The delay functions relate the waiting times to the arrival and service time distributions. The basic concept of this method is to identify the parameters of the interarrival and interdeparture time distributions for each lock, and then estimate the implied waiting times.

To estimate delays in a queuing system, we need to know the means and variances of the interarrival, interdeparture and service time distributions. For series of G/G/1 queues and bidirectional servers, a difficulty arises in identifying the variances of interarrival and interdeparture times. Because the interarrival times at each lock depend on departures from both upstream and downstream locks, and the variances of interarrival times cannot be determined from one-directional scans along a series of queues. To overcome such complex interdependence, an iterative scanning procedure is proposed. The core concept is to decompose the system into individual locks and then sequentially analyze each of those locks. At each lock, the two arrivals from both directions are first combined into an overall arrival distribution and then split into two-directional departure distributions.

The algorithm is initiated by scanning along waterways from either direction, sequentially estimating the interarrival and interdeparture time distributions for each lock. Initially assumed values for the variances of interdeparture times from the opposite direction must be provided for the first scan. Then, the scanning direction is reversed and the process is repeated, using the interdeparture time distributions for the opposite direction estimated in the previous scan. Alternating directions, the scanning process continues until the relative difference in the preselected convergence criteria stays within preset thresholds through successive iterations. Waiting times at locks can be computed in every iteration (and then used as convergence criteria) or just once after all iterations are completed.

Arrival Processes

The mean and standard deviation of interarrival times are estimated in two steps. First, the means and standard deviations of directional interarrival times at a particular lock are estimated from the interdeparture time distributions of the adjacent locks. If flows are conserved between locks and if the V/C ratio is less than 1.0, such relations are represented in Eq. 3:

$$\bar{t}_{aji} = \bar{t}_{djk} \quad \begin{cases} k=i-1, \text{if } j=1 \\ k=i+1, \text{if } j=2 \end{cases} \tag{3}$$

where

$\bar{t}_{aji}$: the average interarrival time for Direction j and Lock i

$\bar{t}_{djk}$: the average interdeparture time for Direction j and Lock k

j : direction index (1 = downstream, 2 = upstream)

Because speed variations change headway distributions between locks, Eq. 4 was developed to estimate the standard deviation of directional interarrival times at one lock.

$$\sigma_{aji} = \sigma_{djk} + \underset{(0.002)}{0.0251}\ln\left(1+\frac{D_{ik}\sigma_{vik}}{\mu_{vik}}\right) \quad \begin{cases} k=i-1, \text{if } j=1 \\ k=i+1, \text{if } j=2 \end{cases} \tag{4}$$

$R^2 = 0.999954$ $\quad$ $n = 107$ $\quad$ $s_e = 0.0586$ $\quad$ $\mu_y = 5.1685$

where

σ_{aji} : standard deviation of interarrival times for Direction j and Lock i

σ_{djk} : standard deviation of interdeparture times for Direction j and Lock k

D_{ik} : distance between Locks i and k

μ_{vik} : average tow speed between Locks i and k

σ_{vik} : standard deviation of tow speeds between Locks i and k

j : direction index (1 = downstream, 2 = upstream)

S_e : standard error of dependent variable

μ_y : mean of dependent variable

This suggests that, theoretically, the standard deviation of directional interarrival times should be equal to the standard deviation of directional interdeparture times plus an adjustment factor depending on the speed distribution and distance.

Second, the overall mean and coefficient of variation of interarrival times for this lock are estimated based on the coefficient of variation of directional interarrival times.

$$\bar{t}_{Ai} = \frac{\bar{t}_{a1i} * \bar{t}_{a2i}}{\bar{t}_{a1i} + \bar{t}_{a2i}} \tag{5}$$

$$\underset{(0.027)}{C_{Ai}^2 = 0.179} + \underset{(0.014)}{0.41}(C_{a1i}^2 + C_{a2i}^2) \tag{6}$$

$R^2 = 0.9188$ $n = 79$ $s_e = 0.0059$ $\mu_y = 0.988$

where

$\bar{t}_{Ai}$: the average interarrival time at Lock i

C_{Ai}^2 : squared coefficient of variation of interarrival times at Lock i

C_{aji}^2 : squared coefficient of variation of directional interarrival times for Direction j and Lock i

In Eq. 6, the coefficients of variation of upstream and downstream interarrival times carry the same weight in estimating the overall variance of interarrival times, since the mean directional trip rates are equal.

Departure Process

The departure module estimates the mean and coefficient of variation of interdeparture times. Based on the flow conservation law, if capacity is not exceeded, the average directional interdeparture equals the corresponding interarrival time:

$$\bar{t}_{dji} = \bar{t}_{aji} \tag{7}$$

The coefficient of variation of interdeparture time is estimated in two steps. First, the coefficient is estimated for combined two-directional departures. Departure processes with generally distributed arrivals and service times are analyzed using Laplace transforms. The following metamodel was eventually developed to bypass the difficulties of determining the variances of the lock idle times:

$$C_D^2 = 0.207+0.795(1-\rho+\rho)+(\rho^2-\rho^2) = 0.207+0.795 = 1.002 \approx 1.0 \quad (8)$$

Next the coefficient of variation of directional interdeparture times is estimated. The following metamodel was developed for this purpose:

$$\underset{}{C_{dji}^2} = \underset{(0.0056)}{0.518} + \underset{(0.0068)}{0.491} C_{aji}^2 \, C_{Di}^2 \quad (9)$$

$R^2 = 0.9710$ $n = 158$ $s_e = 0.013$ $\mu_y = 0.9164$

where

C_{dji}^2: squared coefficient of variation of directional interdeparture times for Direction j and Lock i

C_{aji}^2: squared coefficient of variation of directional interarrival times for Direction j and Lock i

C_{Di}^2: squared coefficient of variation of interdeparture times for Direction j and Lock i

Delay Function

The delay function is intended to estimate the average waiting time at a lock. By applying Marshall's formula for the variance of interdeparture times an exact solution for the average waiting time W, was obtained as follows:

$$W = \frac{\sigma_A^2 + 2\sigma_S^2 - \sigma_D^2}{2\bar{t}_A(1-\rho)} \tag{10}$$

where

W: the average waiting time

σ_A^2: variance of interarrival times

σ_S^2: variance of service times

σ_D^2: variance of interdeparture times

t_A: average interarrival time

ρ: volume to capacity ratio

In this delay function, the average waiting time increases as the variance of interarrival and service times increase and decreases as the variance of interdeparture times increases. The average waiting time approaches infinity as the V/C ratio approaches 1.0.

Comparison of Simulated and Numerical Results

To validate the numerical method, its results were compared to the results of the previously validated simulation model. Various system configurations were compared, including a relatively large 20-lock system.

The parameter values for this test system (e.g. means and standard deviations of input distributions and distances between locks) were obtained from random number generators, except for traffic volumes, which were assumed to be 10 tows/day in each direction throughout the system. Table 3 shows the input parameters and a comparison of waiting times, which are the output variables of greatest practical interest. It can be seen that the numerical model estimates aggregate waiting times within 8% of those simulated. At individual locks, the percent errors are slightly greater but within 10%. In its current form, the modeling approach does not consider possible diversion to other modes on the basis of excessive delay. However, the model might be applied iteratively with a demand reestimation model.

3.3 Project Sequencing and Scheduling

Either the simulation model or the metamodels may serve as a project evaluation tool. That is, both are able to provide delay estimates for a system of locks for different combinations of proposed lock improvements (i.e. any measure that physically or effectively increases the capacity of a lock). This is the basis for estimating the benefits associated with such improvements. The choice should be based on a tradeoff between precision for complete lock operations (favoring simulation) and computational efficiency (favoring the metamodels). Thus, the metamodels may be used for preliminary screening and the simulation for the final detailed evaluation.

The next step in the investment planning methodology is a technique whereby the permutations of investment sequences may be efficiently searched and a corresponding optimal schedule found.

The proposed approach for searching the solution space of possible project permutations represents the solution space in two dimensions and applies a heuristic search algorithm in selecting the preferred sequence. Given a system cost evaluation function for interdependent projects $g(\mathbf{X},\mathbf{Y})$, the selection and sequencing problem may be represented in two dimensional space. The function $g(\mathbf{X},\mathbf{Y})$ incorporates both benefit and cost factors into a generalized cost while accounting for project interdependencies where $\mathbf{X}$ is a vector of delay variables and $\mathbf{Y}$ represents a particular combination of projects.

Assuming that each set of projects may be viewed as a system generating a common time-dependent output, then a two dimensional representation is quite feasible. For the lock rehabilitation problem, the costs associated with a given combination of projects in a given time period t, may be written as

$$(SC)_{tY} = C_Y + g\ (X(\lambda(t)),Y)(OC) \qquad \textbf{(11)}$$

where C_Y is the total capital cost of construction for the set of projects Y. The term $g(X(\lambda(t)),Y)$ represents the delay, and corresponds to the function(s) obtained from some interdependent evaluation, e.g. from a simulation model, while OC is the opportunity cost of delay (which may be either a constant or a function of time). Evaluating SC at different levels of output for a combination of projects Y, defines a curve with annual system costs SC_Y on the vertical axis and output level, λ, on the horizontal axis. Repeating for different values of Y (i.e. different project combinations) produces a family of curves. By always choosing the lowest cost curve for any given output level λ, i.e. by choosing the "lowest envelope" of the curves in Figure 2, a sequencing and scheduling decision path is defined. Because the output is assumed to be time dependent, the horizontal axis may also represent time periods, e.g. years. Output and time may be linked through a demand function, $\lambda(t)$.

Consider an example with interdependent projects A, B, and C. Figure 2 shows a family of system cost (SC) curves corresponding to the possible combinations of these three projects. Note that in general, combinations involving only one project are preferable (lower SC) for low levels of volume (thus earlier in the horizon stage), and become less preferable as volume increases. Under this representation, one combination is preferred to another at a given output level (or time period) if its corresponding curve lies above the other.[1]

In the example depicted in Figure 2, the selection and sequence of projects is dictated by the lowest "envelope" defined by the curves. This lowest envelope corresponds to the minimization of the time integral of the system cost for feasible expansion paths. Here, all three projects would be accepted if the

[1]Although the convex and monotonically increasing properties of the curves in Figure 1 are likely to occur for costs with a delay component, they are not a prerequisite for the methodology.

volume level is expected to eventually exceed Q_2. We see also that the sequence of projects should be A, B, C; this is because Curve A lies below B and C, and AB lies below AC in the relevant regions. Project A is preferred up to volume level Q_1 at the same time Project B should be implemented since Curve AB falls below Curve A. At volume level Q_2, Project C should be added to A and B, thus implementing Combination ABC.

Unfortunately not all such families of curves can be interpreted as easily as Cases 1 and 2. Consider a second case shown in Figure 3 where Curves A and AB are unchanged but the others are different. Here, Curves AB and AC intersect each other before intersecting Curve ABC. It cannot be stated a priori whether Combination AB or AC should be selected on the expansion path between A and ABC. One would expect that if Area 1 is greater than Area 2, then Combination AB is preferred to AC and Project B should precede Project C on the expansion path. Areas 1 and 2 correspond to the difference savings when integrating over Paths A-AB-ABC and A-AC-ABC, respectively.

Scheduling

Under the assumption that the benefits associated with a given combination of projects in some period vary only with the output of the system in that period, the start dates of the projects do not affect the system costs. Thus the SC curves for a project combination depend only on the presence, rather than start times, of particular projects in that combination. The implications in the context of waterways are that the capital cost of construction, operating and maintenance costs, and benefits from reduced delays are not affected by the age of the locks at any given time (i.e. by project start dates) but only by the volume of traffic using the locks. This assumption is very reasonable for the capital costs, but somewhat simplifies the operating and maintenance costs. The assumption is also reasonable for delay benefits although it neglects the effect of long term economic changes induced by the presence and performance of waterway investments.

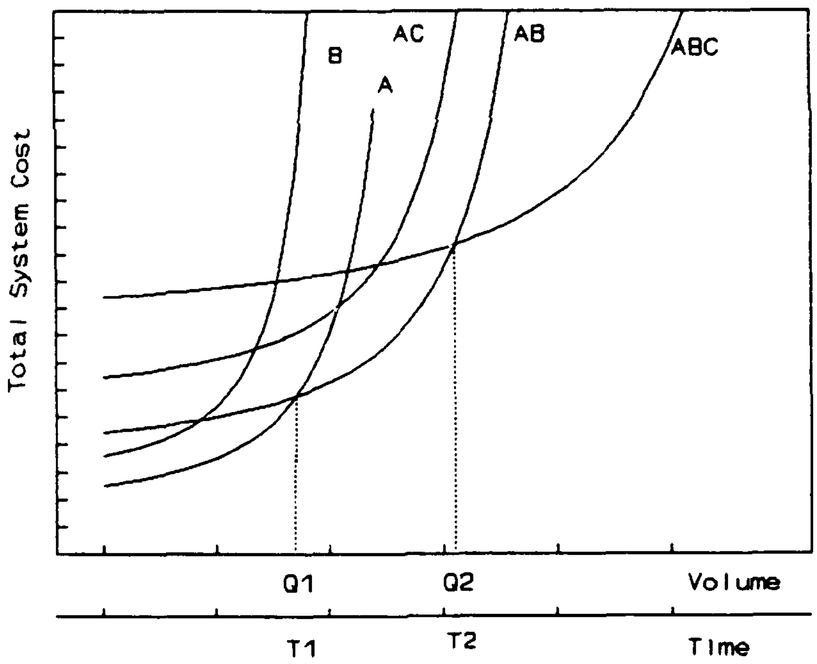

Figure 2 Plot of System Cost for 3 Interdependent Projects (Case1) Incorporating a Budget Constraint

In structuring the budget constraint, it will be assumed that funds not spent in a given period will be available in subsequent periods. Under this assumption, budget limitations have the effect of delaying the earliest feasible start date of a given project combination, just as they limit the earliest start of an individual project. Consider the small example of two projects A and B. In constructing the Curves A, B, and AB, the infeasible portion must not be included. Figure 4 illustrates that Combination A is not financially feasible until time T_1 corresponding to output Q_1. Combination AB is not feasible until time T_2. The three possible expansion paths are then as follows:

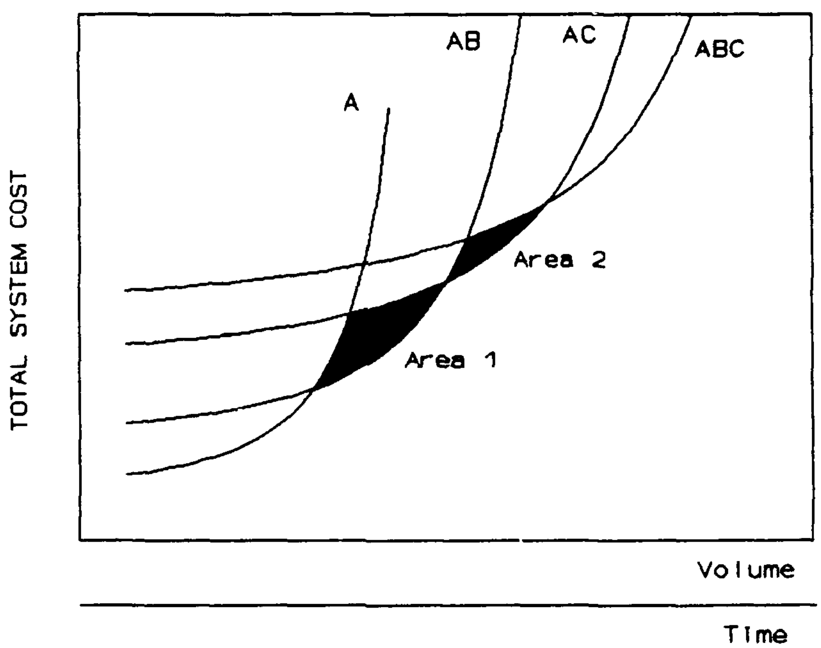

Figure 3 Plot of System Cost for 3 Interdependent Projects (Case2)

1. start A at time T_1 and B when Curves A and AB intersect

2. start B immediately and A when Curves B and AB intersect.

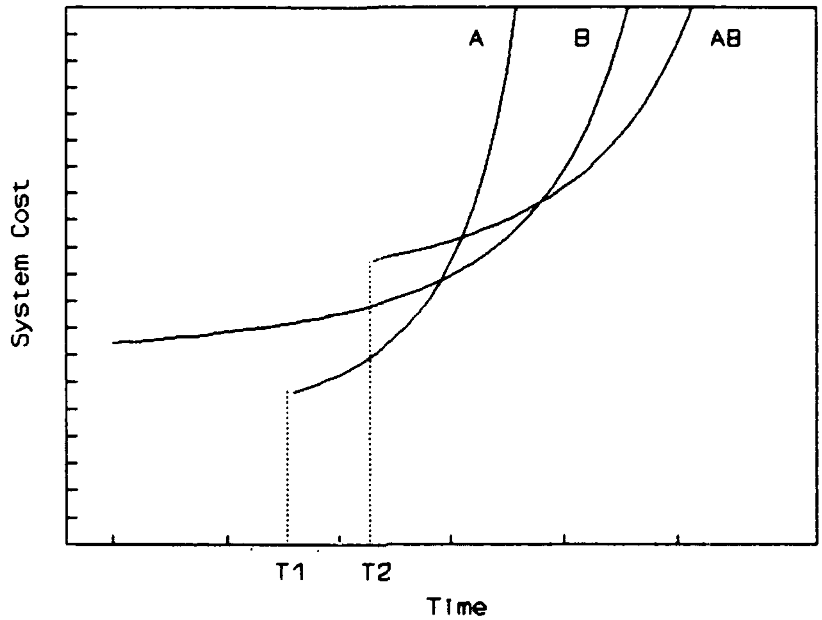

Figure 4 Incorporating a Budget Constraint

In the validation systems of four and six locks were used to compare the solution from the algorithm with that obtained through exhaustive enumeration[2]. In these four and six lock experiments the optimal answer was found by the algorithm in 93.3 and 95 percent of the cases. In the suboptimal cases, the cumulative costs were within 1% of those of the optimal sequence.

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[2]Conducting such tests on larger systems is not possible because the optimal solution cannot be determined for comparison with the solution obtained from the sequencing methodology.

3.4 Cash Flow Analysis

The output of sequencing and scheduling algorithm is the order in which the projects are to implemented and the project start times (i.e. the time construction is complete and the facility is returned to full operation). Unfortunately, the implementation of construction schedules are not without uncertainties. Often, projects may be delayed due to funding interruptions, technical complications, cost overruns or other unforseen conditions. Such delays and overruns can be binding on the Inland Waterway Trust Fund (IWTF). For example, if soil and geological surveys incorrectly assess the type of foundational rock, a project might be interrupted to permit further engineering and design. For this reason, it is helpful to have a methodology for evaluating the financial sensitivities to changes in project costs and schedules.

Such a methodology was developed and programmed for the Corps of Engineers to conduct sensitivity analysis of the IWTF with respect to numerous scheduling and budgeting parameters. The primary computational objective behind the methodology is to reveal the resulting Trust Fund balance profile over a specified planning horizon. The methodology allows for the inclusion of the numerous factors in obtaining the cash flow profile of the IWTF, for example:

1. project sequence and start dates,

2. distribution of project costs over the construction period,
3. duration of the construction period,
4. length of any project interruptions,
5. interest rate accrued on unspent sums over the planning horizon,
6. fuel consumption rates over the planning horizon, and
7. fuel tax rates over the planning horizon.

The computer program that implements the cash flow analysis consists of four modules and a comprehensive user interface. The scheduling module provides utilities for controlling project specific parameters such as project start time, construction duration, and interruptions. The expenditure module considers four basic Trust Fund parameters: distribution, federal matching share, inflation, and base year for discounting. This module provides for three types of expenditure distributions (normal, uniform, and user defined. The revenue module incorporates the fuel tax, fuel consumption and account interest rates to determine the total revenues available in each time period. The output module provides a summary table of the Trust Fund balances and a host of graphic utilities and summary statistics. The computer program has been successfully applied to analyze the sensitivity of the Trust Fund balance to many of the possible uncertainties.

4. Additional Applications of the Methodology

Various applications can be envisioned for this entire methodology or for some of its components. These included the following:

1. Estimation of lock delays under various conditions such as congestion levels, stall patterns, traffic mix, operational improvements, major capacity improvements, and closures for maintenance.

2. Computer evaluation of various lock operating options such as chamber assignment selection for tows, grouping of vessels in chambers, use of helper boats, priorities among vessels, and platooning (m-up-n-down).

3. Investment planning and programming including selection and timing of new projects and smaller scale improvements under financial constraints.

4. Improved management decisions for tow operators, e.g. optimizing fleet schedules and operating speeds under various levels of lock congestion and unreliability.

5. Improved management decisions for shippers, e.g. inventory policies, mode choice and facility location decisions.

6. Improved demand forecasting, based on an improved estimate of future service levels. Beyond such waterway applications, it appears that the approximation methods for queuing networks may be applied in other types of systems such as road networks,

communication networks, manufacturing plants, and parallel computer processors. The algorithm for scheduling interdependent projects should have even wider applicability.

5. Conclusions and Extensions

A fairly comprehensive methodology has been developed for evaluating and scheduling waterway system improvements. Some of the elements may be separately used in several other important applications. Some relatively complex aspects of the waterway system, such as the interactions among delays at adjacent locks, the effects of relatively rare lock failures on delays, and the effects of reliability and congestion on tow operating decisions and shipper inventory policies can be analyzed with this methodology.

Further research would be desirable in several areas, including the following:

1. improved microsimulation components to analyze, in greater detail, various lock operating options,
2. improved metamodels for the approximation of operating characteristics at multiple chamber locks,
3. hybrid model switching automatically between simulation and metamodels depending on required model sensitivity ,
4. new variants of the scheduling algorithm, which trade

computation time for improved solutions,

5. connections to a model that predicts equilibrium demand over time in a multimodal network.

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METAMODELS FOR ESTIMATING WATERWAYS DELAYS THROUGH A SERIES OF QUEUES

by

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METAMODELS FOR ESTIMATING WATERWAY DELAYS

THROUGH SERIES OF QUEUES

by

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ABSTRACT

A numerical method has been developed for estimating delays on congested waterways. Analytic and numerical results are presented for series of G/G/1 queues, i.e., with generally distributed arrivals and service times and single chambers at each lock. One or two-way traffic operations are modelled. A metamodelling approach which develops simple formulas to approximate the results of simulation models is presented. The structure of the metamodels is developed from queueing theory while their coefficients are statistically estimated from simulation results.

The numerical method consists of three modules: (1) delays, (2) arrivals and (3) departures. The first estimates the average waiting time for each lock when the arrival and service time distributions at this lock are known. The second identifies the relations between the arrival distributions at one lock and the departure distributions from the upstream and downstream locks. The third estimates the mean and variance of interdeparture times when the interarrival and service time distributions are known.

The method can be applied to systems with two-way traffic through common bi-directional servers as well as to one-way traffic systems. Algorithms for both cases are presented. This numerical method is shown to produce results that are close to the simulation results.

The metamodels developed for estimating delays and variances of interdeparture times may be applied to waterways and other series of G/G/1 queues. These metamodels for G/G/1 queues may provide key components of algorithms for analyzing networks of queues.

INTRODUCTION

Inland waterway transportation is quite important in the U.S. and other regions, especially for heavy or bulky commodities, since it is inexpensive, energy efficient and safe. Most U.S. waterways consist of stepped navigable pools formed by dams across natural rivers. The lock structures used to raise or lower vessels between adjacent pools constitute the major bottlenecks in the waterway network [19] and generate extensive queues. Some locks have only one chamber, while others may have two parallel chambers whose characteristics may differ. The most common chamber sizes are 110*1200 (i.e., 110 ft wide, 1200 ft long) and 110*600. Each chamber size can accommodate a limited number of barges at one time. For example, a 110 ft

* 1200 ft chamber can accommodate at most 17 standard barges plus a towboat while a 110 * 600 chamber can accommodate at most 8 standard barges plus a towboat. If a tow has more barges than the chamber can accommodate, it must be disassembled into several pieces (called "cuts") to move through the chamber and must later be reassembled. Therefore, the service time distributions depend on chamber size and tow size distributions. Sometimes, chambers will be out of service (i.e., "stalled") due to various causes such as freezing, accidents, and mechanical failures.

A reliable and efficient method for estimating lock queueing delays is essential for evaluating and scheduling waterway investments. Unlike pure queueing theory, simulation methods can be used to model the complexities of waterway operations. However, when many interdependent lock investment proposals are considered, their selection and scheduling becomes a large combinatorial problem [18] and simulation may require too much computer time for practical applications. Hence a numerical approximation method which combines queueing theory and simulation results is proposed here for estimating delays through series of waterway queues.

Figure 1 shows a simple diagram of a lock queueing system. Locks are the servers and tows are customers waiting to be served by locks. In the lock queueing system tows from both directions, upstream and downstream, share the same lock servers, while in most other queueing systems the servers are exclusively one-directional. In this paper, the term "two-way traffic operations" characterizes the lock queueing system while "one-way traffic operations" describes a more general queueing system.

Arrival and service time distributions at locks are fairly complex. Carroll [2] and Desai [8] found that service times are not exponentially distributed, and arrivals are not Poisson distributed. Other standard distributions have been tested for the present study, without consistent success. Thus, empirical distributions (specified for 50 intervals) are used here for simulation while general distributions, described only by their means and variances are used for queueing models. Although isolated locks with a single chamber may be modeled as G/G/1 queueing systems, locks with two parallel chambers may not be treated simply as G/G/2 queueing systems unless these parallel chambers are identical.

The lock service time distributions are affected by the chamber assignment discipline at locks with two dissimilar chambers. There the "main" chamber is larger than the "auxiliary" chamber

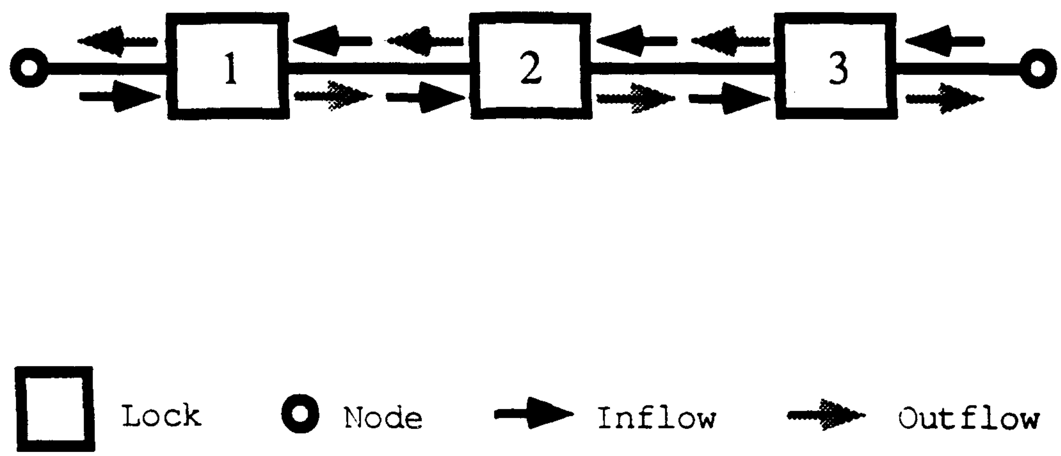

Figure 1. Lock Queueing System

and can accommodate without disassembly large tows which might require several cuts and far larger service times to move through the auxiliary chamber. However, if the same number of cuts is required through either chamber, the auxiliary chamber may provide faster service. Generally, an auxiliary chamber has the same width as a main chamber (110 ft), but a shorter length (360 or 600 rather than 600 or 1200 ft). When a tow requires the same number of cuts in either chamber, it typically takes less time to move through the shorter auxiliary chamber. Therefore, lock service time distributions are dynamic and depend on the chamber assignment discipline.

Considerable interdependence may exist among locks in a series. The departure distributions differ from the arrival distributions since the service time distributions change the tow headways. The departures from one lock usually affect the arrivals at the next lock. Therefore, it is improper to assume such locks are independent. That drastically impairs the applicability of stochastic queueing theory. The interdependence among locks increases the difficulty in estimating delays for the lock queueing system since it is necessary at each lock to identify the interarrival time distributions of flows from adjacent locks.

Two-way traffic operation through common servers complicates the interdependence of lock delays and precludes the use of some otherwise interesting queueing models. Delays are determined by the arrival distributions and service time distributions. It is much more difficult to identify the arrival distributions for two-way traffic systems than for one-way traffic systems. The arrival distribution at one lock is affected by departures from both upstream and downstream locks, while departures from this lock also affect the arrivals at upstream and downstream locks. For example, in Fig. 1 the arrivals at Lock 2 would be affected by the departures from Locks 1 and 3. The departures from Locks 1 and 3 toward Lock 2 are highly correlated with the arrivals at 1 and 3 from 2. At least some of the arrivals at 1 and 3 represent departures from 2. Hence, the arrival distributions of these three locks are interdependent. Thus, two-way traffic operation greatly complicates the estimation of the two arrival distributions at each lock.

The arrival distributions cannot be determined directly from the departure distributions of the adjacent locks since tow speed variations between locks alter the arrivals. Thus the arrival distributions at one lock are affected not only by the departure distributions from adjacent locks, but also by the distances and speed distributions between these locks.

Random failures, which in inland waterways are called stalls, contribute significantly to the difficulties in estimating delays. Stalls, which interrupt lock operations and thereby increase delays, are relatively rare compared to other events. Their occurrence is very difficult to predict. Thus, Kelejian's efforts to model stalls and stall durations have not yet yielded strong results despite the rigorous statistical methods employed [11].

The purpose of this research is to estimate delays for a realistic lock queueing system, assuming the interarrival time distributions at the system boundaries and the lock service time distributions are known. The difficulties in estimating delays for such a lock queueing system are summarized as follows:

1. Arrival and service time distributions are generally distributed.
2. Parallel chambers are not identical.
3. Service time distributions would be affected by the chamber assignment discipline.
4. Considerable interdependence exists among a series of locks.
5. Two-way traffic operation through bi-directional chambers complicates the analysis.
6. The arrival distributions depend on not only the departures from previous locks but also on the distances and speed distributions between locks.
7. Stalls increase the means and variances of delays.

The available analytic solutions for estimating delays in G/G/1 queues are quite inadequate. Kleinrock [13] suggested an approximation solution for a G/G/1 queue with heavy traffic. In fact, this approximation solution is an upper bound for average waiting times in G/G/1 queues. It works well when the volume to capacity (V/C) ratio approaches 1.0 but generates significant errors in estimated delays for low to medium traffic levels.

Bertsimas [1] derived an exact delay solution with mixed generalized Erlang distributed arrivals and service times. This result could be applied to more realistic situations than Poisson arrivals and exponential service times. However, without a departure function, this result is difficult to extend to a series of locks.

DeSalvo [7] and Wilson [23] tried to estimate lock delays by treating locks as M/M/1 and M/G/1 queues, respectively. These two models did not account for interdependence among locks and their assumptions significantly limited their applicability.

System simulation models to analyze lock delays and tow travel times have been developed by Howe [10] and by Carroll and Bronzini [3]. These modelled the waterway systems in considerable detail and required considerable input data and computer time. Both models assumed Poisson distributions for tow trip generation and did not account for stalls. However, simulation models should, at least in principle, be able to represent the complexities of traffic on waterway networks much better than analytic queueing models. Thus, a more efficient simulation model able to accommodate stall effects was desirable for this work.

Such a simulation model was developed in the early stages of this study [6]. This model can accommodate generally distributed trip generations and service times. It can also evaluate the stall effects. Although this simulation model requires only a few seconds to a few minutes on PS/2 computer for each run, that is still hardly affordable for direct application in large combinatorial network investment problems.

To avoid the computational expense of simulation when evaluating numerous combinations and schedules of network improvements, a metamodelling approach [16, pp. 679-689] was developed to approximate the results of the simulation model. The complete methodology used in this study consists of (1) developing and validating a simulation model to represent waterway networks with queues at locks, (2) formulating functions developed from queueing theory for delays through series of locks, (3) statistically estimating the parameters of these functions using simulation results, and (4) employing an iterative sequential scanning procedure to estimate interarrival and interdeparture time distributions lock by lock until results converge at each lock. Thus, relatively simple equations may serve as a proxy for the simulation model.

Basically, the numerical method is a decomposition model. It decomposes systems of queues into separate queueing stations. The analysis of each queueing station is decomposed into three steps, namely arrivals, departures and delays. Such decomposition techniques have been widely applied for analyzing networks of queues [4,14,15,20,21,24]. These previous studies show that it is usually sufficient to approximate all the flows in networks of queues by renewal processes characterized by two parameters (the mean and the variance).

The proposed numerical method may be applied not only to the lock queueing system, but also to some other series of queues. The departure processes module and delay function may be applied to networks of G/G/1 queues. Further research in estimating interarrival time distributions with multiple unequal inflows would be necessary for extending the numerical method to general networks of queues which may have inflows and outflows at any node.

1. Simulation Model

A detailed discussion of the simulation model developed for this work is included in Dai and Schonfeld [6]. A brief description of the simulation model and its validation is provided below.

1.1 Data Base

The simulation model was developed on the basis of PMS (lock Performance Monitoring System) data collected since 1975. This data base includes very detailed information on traffic through the locks as well as physical aspects of lockages [9]. It is very useful for understanding and quantifying waterway characteristics, such as lock operations, arrival distributions, service times distributions, tow size distributions and stalls.

1.2 Model Features

The simulation model is programmed in Fortran-77 which provides great flexibility in modelling. Basically, it is a stochastic, microscopic and event-scanning simulation model which can handle any distributions for trip generation, travel speeds, lock service times and tow sizes. These distributions can be specified for each interval in tables or by standard statistical distributions. Currently, normal distributions are used for travel speeds while general distributions based on empirical observations are used for other input variables. Tows are allowed to overtake other tows. A FIFO (First-In-First-Out) service discipline is currently employed. This model simulates two-way traffic through common servers and accounts for stalls.

1.3 Validation

To check the logic of this simulation model, its results are first compared to theoretical (but very well established) results from queueing theory. This also checks the model's ability to represent general series of queues. The model's results are then compared with observed data to demonstrate how closely the model represents real systems and verify its ability to simulate the special features of waterways.

The model's predicted waiting times at a single lock are compared with those obtained from queueing theory when arrivals are Poisson distributed and service times are generally distributed. The validation is conducted for a variety of volume/capacity (V/C) ratios ranging from .0471 to .8934. To reduce the simulation variance each result is obtained by averaging the output from 30 independent simulation runs. To insure results are compared for a steady state, each simulation run discards the first 10,000 tow waiting times and collects the next 12,000 values for computing the average waiting time. The results are shown in Table 1. They confirm that the simulated and theoretical average waiting times are extremely close. Such results verify that the overall mechanism of the simulation model is correct. They also show that generally distributed service times are generated satisfactorily in the simulation model. That is reassuring since the same logic is also used to generate generally distributed interarrival times for G/G/1 queues and, ultimately, to develop metamodels for series of G/G/1 queues.

Table 1 Comparison of Theoretical and Simulated Results for a Single Lock Queue (M/G/1)

Interarrival Times Avg (hr)	Interarrival Times Var (hr^2)	Service Times Avg (hr)	Service Times Var (hr^2)	V/C	W_{sim} [1] (hr)	W_t [2] (hr)	Deviation (%)
.8880	.7886	.7933	.3188	.8934	4.9516	5.0059	-1.09
.8880	.7886	.6701	.2274	.7546	1.5575	1.5522	0.34
.8880	.7886	.5025	.1280	.5659	0.4926	0.4935	-0.19
.8880	.7886	.2930	.0435	.3300	0.1082	0.1087	-0.46
.8880	.7886	.0418	.0009	.0471	0.00155	0.00156	-0.64

1 W_{sim}: average waiting times from simulation
2 W_t: average waiting times from queueing theory

The simulation results are then compared with the observed data (from January 1987) at Locks 22, 24, 25, 26 and 27 on the Mississippi River. At that time, Locks 22, 24 and 25 had single 600 ft long chambers. Locks 26 and 27 had two chambers each (600 ft and 360 ft long at Lock 26, 1200 ft and 600 ft at Lock 27). The validation results are summarized in Tables 2, 3 and 4. Each result is averaged from 80 independent simulation runs. The initial condition for simulation is assumed to be an empty system, which is consistent with the observed condition for this system in winter.

Table 2 shows that the simulated average waiting times for each lock and for the whole series of locks are close to those observed except at Lock 25. The observed data also show that tows sometimes were kept waiting at Lock 25 even when the chamber was idle. Such operation is somewhat unusual. Therefore, no direct comparison of average waiting times at Lock 25 is appropriate. Tables 3 and 4 also show that the simulation model represents the real system quite well.

Table 2 Comparison of Simulated and Observed Average Waiting Times

Lock	W_{sim}[1] (min)	W_{obs}[2] (min)	Difference (min)	σ_{sim}[3] (min)	σ_{obs}[4] (min)	95% Confidence Interval (min)
22	4.09	3.73	0.36	2.87	15.19	3.49
24	6.12	6.36	0.24	4.14	29.74	6.72
25	4.49	10.94	6.45	4.50	19.19	-[5]
26	119.40	130.99	11.59	28.91	271.11	60.73
27	36.49	34.43	2.06	30.37	106.42	23.92

1 W_{sim}: simulated average waiting times
2 W_{obs}: observed average waiting times
3 σ_{sim}: standard deviation of simulated waiting times
4 σ_{obs}: standard deviation of observed waiting times
5 The comparison is not appropriate.

Table 3 Comparison of Chamber Volumes

Lock	Chamber	Vol_{sim}[1] (tows/month)	Vol_{obs}[2] (tows/month)	Difference (tows/month)	95% Confidence Interval (tows/month)
22	1	44.61	45	0.39	1.88
24	1	56.00	56	0.00	2.44
25	1	51.40	52	0.60	2.13
26	1	275.20	265	10.20	2.83
26	2	155.96	167	11.04	3.81
27	1	390.95	389	1.95	10.25
27	2	306.73	306	0.73	10.62

1 Vol_{sim}: simulated volumes
2 Vol_{obs}: observed volumes

Table 4 Comparison of Cut Volumes

Lock	Chamber	Cuts[1]	Vol_{sim}[2] (tows/month)	Vol_{obs}[3] (tows/month)	Difference (tows/month)	95% Confidence Interval (tows/month)
22	1	1	30.35	31	0.65	1.47
22	1	≥2	14.26	14	0.26	-
24	1	1	39.76	40	0.24	1.76
24	1	≥2	16.24	16	0.24	-
25	1	1	35.88	37	1.12	1.65
25	1	≥2	15.52	15	0.52	0.97
26	1	1	75.46	74	1.46	2.04
26	1	≥2	199.74	191	8.74	3.25
26	4	1	137.25	147	9.75	2.87
26	4	≥2	18.71	20	1.29	-
27	1	1	390.95	389	1.95	10.25
27	4	1	269.05	265	4.05	5.59

1 "Cuts" are subsets of barges into which tows are subdivided for passage through lock chambers
2 Vol_{sim}: simulated volumes
3 Vol_{obs}: observed volumes

Each simulation run takes a few seconds to a few minutes on a personal computer, depending on traffic volumes, duration of simulation periods, network size, and other factors. Despite that, simulation time becomes expensive for evaluating large combinatorial investment scheduling problems. For example, when there are n=20 possible investment projects, it is necessary to simulate 2^{20} combinations to make the best decision. $30*2^{20}$ separate simulation runs are then required if each performance measure is based on the average over 30 independent replications. Furthermore, the project combinations may have to be evaluated over several time periods. Therefore, as n increases, direct evaluation by simulation becomes very expensive.

A metamodelling approach is proposed to overcome the computational requirements of simulation. A simulation model is then treated as a function with unknown explicit form which turns input parameters into output performance measures. The metamodelling approach provides a method to develop simple formulas to approximate this function. However, the structural forms of these formulas are quite important and not intuitively obvious. In queueing systems applications, the structural forms should be based as closely as possible on queueing theory.

2. Numerical Method

2.1 Methodology

In this study, a numerical method has been developed for estimating delays through a series of queues. This method was originally developed for systems with bi-directional servers. With a few simplifications, this method can be adapted for the more generally encountered systems with one-directional servers. The numerical method decomposes the entire networks of queues into each individual queueing stations. The method consists of three major modules: arrival processes, departure processes, and delay functions. Arrival processes at a particular lock depend on the departure distributions from the upstream and downstream locks. The departure processes depend on the interaction among the arrival distributions and service time distributions at one lock. The delay functions define the relations among waiting times, arrival distributions and service time distributions. The basic concept of this method is to identify the parameters of the interarrival and interdeparture time distributions for each lock, and then estimate the implied waiting times. To be more concerned about the dependence among successive arrivals, the authors also test the correlation of successive arrivals in the waterway systems. The results show that the correlation among successive arrival intervals is very small. Most are close to 0 and no more than 0.2 or less than -0.2. Currently, the following assumptions are used in the numerical method:

1. Arrivals and service times are generally distributed.
2. A single service time distribution applies to both directions of passage through a chamber.
3. All traffic units are identical and have service times at a lock chamber governed by a single probability distribution.
4. Each lock has one chamber.
5. Inflows and outflows occur only at the two end nodes of a series of locks.
6. The average upstream volumes are equal to the downstream volumes.
7. The long-term volume to capacity ratio (V/C) is less than 1.0 at every lock.
8. Idle times and service times are independent.

It should be noted that Assumptions 2, 3, 4, 5, 6 and 7 are only applicable to the numerical method. The simulation model is not limited by those assumptions. The numerical method can provide a quick and inexpensive approach for the analysis of lock delays. However, Assumptions 4, 5, 6 and 7 limit fairly significantly the applicability of the currently developed numerical method and necessitate the substitution of the simulation model when significant deviations from those assumptions must be considered. With some extensions to the numerical method expected in the near future, Assumptions 4 and 5 may be eliminated. Assumption 6 could be relaxed even though it is usually realistic for waterways.

The general service time distribution (Assumption 1) may be adjusted to incorporate the frequency and duration of random failures provided such stalls depend mostly on traffic volumes rather than the passage of time. Although the simulation model underlying the metamodels has only been tested with a First-In-First-Out (FIFO) service discipline, the average delay estimated by a metamodel should not depend on the discipline.

Each module of the numerical method consists of one or more metamodels. The procedures used in developing each metamodel are summarized as follows:

1. Use queueing theory to identify the input (independent) variables which will affect the output measures (dependent variables) and to propose functional relations between the input variables and the output measures with appropriate structure form.
2. Plot the relevant output measures (dependent variables) versus the input (independent) variables. This step helps confirm the relations between the output measures and the input parameters.
3. Compute Pearson correlation tables. This step confirms correlations between selected dependent variables and the independent variables and help avoid multicollinearity problems among the independent variables.
4. Estimate parameters for the proposed functional relations (or "metamodels") and select the preferred metamodel. The coefficient of determination (R^2) is used to compare the explanatory power of the alternatives. In general, the metamodel with R^2 closest to 1 is preferred since it best accounts for the variation of dependent variables. It is also important to test if the independent variable is significant in explaining the dependent variable's variation.
5. Perform residual analysis to check whether this metamodel violates certain regression assumptions, such as normality and homoscedasticity, and to detect outliers. The residual analysis should include the property-analysis and the graphical analysis of residuals. The basic residual properties to be examined include the mean, variance, skewness, and kurtosis.

The graphical analysis is the most direct and revealing way to examine a set of residuals. The residuals can be displayed in one or two dimensions. The useful one-dimensional plot of residuals includes histograms (or stem-and-leaf plots), schematic plots, and normal probability plots [12]. The two-dimensional plot examines the relationships of the residuals to either dependent or independent variables and is useful for identifying violation of regression assumptions such as independence of residuals [12].

The variables and their corresponding ranges which were used in developing the metamodels are listed in Table 5.

Table 5 Ranges of Variables Used to Develop Metamodels

Variable	Minimum	Maximum	Description
C_A	0.91	1.01	Coefficient of variation of interarrival times
C_D	0.63	1.01	Coefficient of variation of interdeparture times
C_S	0.58	0.90	Coefficient of variation of service times
D	4.00	156.00	Distance between locks (mi)
W	0.03	18.50	Average waiting time (hr/tow)
λ	0.04	1.67	Average trip rate (tows/hr)
μ_v	75.60	295.00	Average tow speed (mi/day)
ρ	0.04	0.97	V/C ratio
σ_A^2	0.35	608.46	Variance of interarrival times (hr^2/tow^2)
σ_a^2	1.38	2457.43	Variance of directional interarrival times (hr^2/tow^2)
σ_D^2	0.17	606.44	Variance of interdeparture times (hr^2/tow^2)
σ_d^2	1.05	2457.51	Variance of directional interdeparture times (hr^2/tow^2)
σ_s^2	0.04	4.34	Variance of service times (hr^2/tow^2)
σ_v	2.22	136.30	Standard deviation of tow speeds (mi/day)

2.2 Arrival Processes

The following two steps are used for estimating the mean and variance of interarrival times:

Step 1. Estimate the means and variances of directional interarrival times at a particular lock from the departure distributions of the adjacent upstream and downstream locks.

If flows are conserved between locks and if the V/C ratio is less than 1, the average directional arrival rates at one lock should be equal to the average directional departure rates from adjacent upstream and downstream locks. Therefore, the average directional interarrival times at that lock should also be equal to the average directional interdeparture times from adjacent upstream and downstream locks. Such relations are represented in Eq. 1:

$$\bar{t}_{a_{j,i}} = \bar{t}_{d_{j,k}} \quad \begin{cases} k=i-1, if\ j=1 \\ k=i+1, if\ j=2 \end{cases} \tag{1}$$

where

$\bar{t}_{a_{j,i}}$: the average interarrival time for Direction j and Lock i

$\bar{t}_{d_{j,k}}$: the average interdeparture time for Direction j and Lock k

j : direction index (1 = downstream, 2 = upstream)

If each tow moves at the same speed, the directional arrival distributions at one lock will be the same as the directional departure distributions at the preceding lock. However, speed variations change headway distributions between locks. Using the approach outlined in Section 2.1, Eq. 2 is developed to estimate the variance of directional interarrival times at one lock.

$$\sigma_{a_{j,i}}^2 = \underset{(.178)}{-.354} + \underset{(.003)}{.998}\sigma_{d_{j,k}}^2 + \underset{(.046)}{.127}\ln\frac{\sigma_{d_{j,k}}^2 * D_{i,k}}{\mu_{v_{i,k}}} + \underset{(.046)}{.162}\ln\sigma_{v_{i,k}} \quad \begin{cases} k=i-1, if\ j=1 \\ k=i+1, if\ j=2 \end{cases} \tag{2}$$

$R^2 = 0.9997 \qquad n = 72 \qquad s_e = 0.1263 \qquad \mu = 12.3459$

where

$\sigma_{a_{j,i}}^2$: variance of interarrival times for Direction j and Lock i

$\sigma_{d_{j,k}}^2$: variance of interdeparture times for Direction j and Lock k

$D_{i,k}$: distance between Locks i and k

$\mu_{v_{i,k}}$: average tow speed between Locks i and k

$\sigma_{v_{i,k}}$: standard deviation of tow speeds between Locks i and k

j : direction index (1 = downstream, 2 = upstream)

s_e : standard error of dependent variable

μ : mean of dependent variable

Standard errors are shown parentheses under the estimated parameters of Eq. 2.

Currently, there is less theoretical basis for Eq. 2 than for the other metamodels developed in this study. This metamodel was developed largely by empirical analysis. The dependent variable was plotted versus possible influential factors, including the variance of directional interdeparture time distributions, the distance between two locks, the average tow speed, and the standard deviation of tow speeds. These plots help confirm the structural form of this metamodel.

It is noteworthy that the coefficient for the variance of directional interdeparture times from the preceding lock is 0.998, which is approximately equal to 1. This suggests that, theoretically, the variance of directional interarrival times should be equal to the variance of directional interdeparture times plus an adjustment factor depending on the speed distribution and distance. The high R^2 value and the small standard error (only about 1% of the mean value of the dependent variable) are also noteworthy.

Step 2. Estimate the overall mean and variance of interarrival times for this lock based on the variances of directional interarrival times which are obtained from the step above.

$$\bar{t}_{A_i} = \frac{\bar{t}_{a_{1,i}} * \bar{t}_{a_{2,i}}}{\bar{t}_{a_{1,i}} + \bar{t}_{a_{2,i}}} \qquad (3)$$

$$\underset{}{\sigma_{A_i}^{2}} = \underset{(0.00379)}{0.0128} + \underset{(0.000083)}{0.1243}(\sigma_{a_{1,i}}^{2} + \sigma_{a_{2,i}}^{2}) \qquad (4)$$

$R^2 = 0.99999$ $\quad n = 108$ $\quad s_e = 0.00109$ $\quad \mu = 3.09753$

where

$\bar{t}_{A_i}$: the average interarrival time at Lock i

$\sigma_{A_i}^{2}$: variance of interarrival times at Lock i

The meaning of Eq. 3 would be clearer if viewed in terms of the average trip rates rather than the average interarrival times. Eq. 3 implies that the overall arrival rate at certain lock is the sum of the average directional arrival rates from upstream and downstream. In Eq. 4, variances of upstream and downstream interarrival times carry the same weight in estimating the overall variance of interarrival times, since directional trip rates are equal here (Assumption 6). Eq. 4 should be reestimated when applied to a directionally imbalanced general network of queues. It may be noted that the arrivals metamodel may estimate the variances of arrival distributions not only at locks but at any distance from a departure point with a known departure distribution (i.e. known mean and variance) such as a junction or port.

2.3 Departure Processes

The departure processes module estimates the mean and variance of interdeparture times. Based on the flow conservation law, if the V/C ratio is less than 1, the outflow rate should be equal to the inflow rate. Therefore, the average directional interdeparture time can be determined from the corresponding interarrival time:

$$\bar{t}_{d_{ij}} = \bar{t}_{a_{ij}} \tag{5}$$

The variance of interdeparture times is estimated in three steps:

Step 1. Estimate the coefficient of variation of interdeparture times. Departure processes with generally distributed arrivals and service times are analyzed by using Laplace transforms. The use of Laplace transforms for derivations in queueing theory (which is quite frequent) is presented in texts such as Kleinrock [13]. Some analytic relations obtained in this study are shown below using the following notation:

Let:

$\bar{t}_A, \sigma_A^2$: mean and variance of interarrival times

$\bar{t}_S, \sigma_S^2$: mean and variance of lock service times

$\bar{t}_D, \sigma_D^2$: mean and variance of interdeparture times

$\bar{t}_I, \sigma_I^2$: mean and variance of lock idle times

ρ : V/C ratio

C_A, C_S, C_D : coefficients of variation for interarrival times, service times, and interdeparture times

$f_A(t)$, $f_S(t)$, $f_D(t)$, $f_I(t)$: probability density functions (pdf) for interarrival times, lock service times, interdeparture times, and lock idle times, respectively

$F_A^*(z)$, $F_S^*(z)$, $F_D^*(z)$, $F_I^*(z)$: Laplace transforms for $f_A(t)$, $f_S(t)$, $f_D(t)$, $f_I(t)$
For example, for interarrival times, the Laplace transform is expressed as

$$F_A^*(z) = \int_0^\infty f_A(t)e^{-zt}\,dt$$

The departure process in a queueing station may be analyzed for two different conditions: with and without a queue. The interdeparture time distribution would be equal to the service time distribution while there are queues waiting for service. However, the interdeparture time would be equal to the sum of the idle time and the service time while there is no queue. Therefore, the Laplace transforms for the interdeparture time distributions can be represented as follows:

$$F_D^*(z)\big|_{with\ queue} = F_S^*(z) \tag{6}$$

$$F_D^*(z)\big|_{without\ queue} = F_I^*(z)F_S^*(z) \tag{7}$$

The probability of having a queue is given by the volume/capacity ratio ρ [13]. Then the probability of not having a queue is $(1-\rho)$. Therefore, the Laplace transform for the interarrival time distribution can be represented by Eq. 8.

$$F_D^*(z)=(1-\rho)F_D^*(z)\big|_{without\ queue}+\rho F_D^*(z)\big|_{with\ queue}=(1-\rho)F_I^*(z)F_S^*(z)+\rho F_S^*(z) \tag{8}$$

The mean of a distribution can be represented by the negative value of the first derivative of its Laplace transform when z equals 0. Therefore, the average interarrival time, service time, interdeparture time, and idle time can be represented by Eqs. 9a to 9d.

$$\bar{t}_A = -\frac{\partial F_A^*(z)}{\partial z}\Big|_{z=0} \tag{9a}$$

$$\bar{t}_S = -\frac{\partial F_S^*(z)}{\partial z}\Big|_{z=0} \tag{9b}$$

$$\bar{t}_D = -\frac{\partial F_D^*(z)}{\partial z}\Big|_{z=0} \tag{9c}$$

$$\bar{t}_I = -\frac{\partial F_I^*(z)}{\partial z}\Big|_{z=0} \tag{9d}$$

The variance can be expressed as the difference between the second derivative of the Laplace transform and the mean. Eqs. 10a-10d express such relations for interarrival time, service time, interdeparture time and idle time distributions.

$$\sigma_A^2 = \frac{\partial^2 F_A^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_A^2 \tag{10a}$$

$$\sigma_S^2 = \frac{\partial^2 F_S^*(z)}{\partial Z^2}\Big|_{z=0} - \bar{t}_S^2 \tag{10b}$$

$$\sigma_D^2 = \frac{\partial^2 F_D^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_D^2 \tag{10c}$$

$$\sigma_I^2 = \frac{\partial^2 F_I^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_I^2 \tag{10d}$$

When z equals 0, the Laplace transform is equal to 1, producing the following relations for interarrival time, service time, interdeparture time and idle time distributions:

$$F_A^*(0)=1 \tag{11a}$$

$$F_S^*(0)=1 \tag{11b}$$

$$F_D^*(0)=1 \tag{11c}$$

$$F_I^*(0)=1 \tag{11d}$$

Combining Eqs. 8, 9b, 9c, 9d, 11b, and 11d yields

$$\bar{t}_D = -\frac{\partial F_D^*(z)}{\partial z}\Big|_{z=0} = -((1-\rho)(-\bar{t}_I-\bar{t}_S)+\rho(-\bar{t}_S)) \tag{12}$$

Due to flow conservation, if the V/C ratio is less than 1, then the average interdeparture time would be equal to the average interarrival time:

$$\bar{t}_D = \bar{t}_A \tag{13}$$

Therefore, Eqs. 12 and 13 can be combined:

$$\bar{t}_D = (1-\rho)(\bar{t}_I+\bar{t}_S)+\rho\bar{t}_S = \bar{t}_A \tag{14}$$

Since $\rho = t_S/t_A$, Eq. 14 yields

$$\bar{t}_I = \bar{t}_A \tag{15}$$

Combining Eqs. 8, 9b, 9d, 10b,10c, 10d, 11b, 11c, 11d, and 15 yields

$$\sigma_D^2 = \frac{\partial^2 F_D^*(z)}{\partial z^2}\Big|_{z=0}-\bar{t}_D^2 = (1-\rho)\sigma_I^2+\sigma_S^2+(\bar{t}_A-\bar{t}_S)\bar{t}_S \tag{16}$$

Dividing Eq. 16 by t_D^2, we can obtain the following relation for the coefficient of variation C_D:

$$C_D^2 = (1-\rho)\frac{\sigma_I^2}{\bar{t}_A^2}+\rho+C_S^2\rho^2-\rho^2 \tag{17}$$

In the special case where the arrival process is Poisson distributed and the service times are exponentially distributed, then due to the memoryless property of the Poisson distribution, the variance of idle times would be equal to the variance of interarrival times. Since the interarrival times for a Poisson process are exponentially distributed and since the mean and standard deviation of an exponential distribution are equal, we can state the following:

$$\sigma_I^2 = \sigma_A^2 = \bar{t}_A^2 \tag{18}$$

$$\sigma_S^2 = \bar{t}_S^2 \tag{19}$$

Therefore, in this special case, Eq. 16 can be simplified to

$$\sigma_D^2 = \sigma_A^2 \tag{20}$$

which is consistent with Burke's theorem [5]. In that theorem Burke proved that when the arrivals are Poisson distributed and the service times are exponentially distributed, then the departures must be Poisson distributed with the same mean and variance as the arrivals.

The main difficulty in estimating the variance of interdeparture times (Eq. 16) when arrivals and service times are generally distributed is determining the variance of the lock idle times. These depend on the way in which the previous busy period terminated. This problem may be bypassed by developing a metamodel for directly estimating the variance of the interdeparture times. Following the approach outlined in Section 2.1, the following metamodel was developed:

$$\begin{array}{l} C_D^2 = 0.396+0.606(C_A^2(1-\rho)+\rho)+1.0002(C_S^2\rho^2-\rho^2) \\ \qquad\quad (0.052) \quad (0.053) \qquad\qquad\qquad (0.0046) \end{array} \tag{21}$$

$R^2 = 0.9977$ n = 120 $s_e = 0.000056$ $\mu = 0.8267$

Eq. 21 was originally developed by using four separate variables: (1) $C_A^2(1-\rho)$, (2) ρ, (3) $C_S^2\rho^2$, and (4) ρ^2. However very high correlations were observed between Variables 1 and 2 and

Variables 3 and 4. Moreover, the coefficients were almost equal for Variables 1 and 2 and for Variables 3 and 4. To avoid multicollinearity problems in developing Eq. 21, Variables 1 and 2 were combined into a single variable while Variables 3 and 4 were combined into a second variable. It is noteworthy that the dependent variable has a standard error of 0.000056, which is only 0.0000677 of its mean.

Since the mean and standard deviation of an exponential distribution must be equal, its coefficient of variation must be 1.0. Thus, for the special case of an M/M/1 queue:

$$C_A^2 = 1 \tag{22}$$

$$C_S^2 = 1 \tag{23}$$

Substituting Eqs. 22 and 23 into Eq. 21, the latter may be simplified as follows:

$$C_D^2 = 0.396+0.606(1-\rho+\rho)+(\rho^2-\rho^2) = 0.396+0.606 = 1.002 \approx 1.0 \tag{24}$$

This result is also consistent with Burke's Theorem [5].

The parameters and structural form of this metamodel are similar to those of Eq. 17 which was analytically derived for G/G/1 queues. In addition, its standard error is extremely tight and it satisfies Burke's Theorem very closely when applied to the special M/M/1 case. Since the general distributions are not specific to waterway applications and are specified only by their mean and variance, this metamodel seems quite reliable and useful for predicting interdeparture time distributions from G/G/1 queues embedded in larger systems, such as series and networks.

Step 2. Estimate the variance of interdeparture times. By the definition of the coefficient of variation, the variance of interdeparture times can be represented as follows:

$$\sigma_D^2 = C_D^2 \bar{t}_D^2 \tag{25}$$

Step 3. Estimate the variance of directional interdeparture times for upstream and downstream traffic. For this purpose the following metamodel was developed:

$$\sigma_{A_{ij}}^{2} = 0.52 + 8.16 \frac{\sigma_{a_{ij}}^{2} + \sigma_{D_i}^{2}}{\sum_{j=1}^{2} \sigma_{a_{ij}}^{2}} \tag{26}$$

(0.08) (0.03)

$R^2 = 0.9971$ $n = 216$ $s_e = 1.0697$ $\mu = 11.9695$

The above metamodel was developed empirically, without a structural form well grounded on queueing theory. Although its R^2 is still quite high, this metamodel accounts for most of the error produced by the complete numerical method. Hence, improvements in this step would clearly be desirable.

2.4 Delay Function

The delay function is intended to estimate the average waiting time at a lock for which the arrival and service time distributions are known. Marchal [17] has proposed the following upper bound for waiting times in a G/G/1 queue:

$$W = 0.5 \frac{\lambda(\sigma_A^{2} + \sigma_S^{2})(1 + C_S^{2})}{(1-\rho)(\frac{1}{\rho^2} + C_S^{2})} \tag{27}$$

This upper bound is exact for M/M/1 and M/G/1 queues.

To develop the delay metamodel, the structures of several upper bounds, including Marchal's, were considered. Since the metamodel with the structure of Marchal's upper bound has the best coefficient of determination (R^2), the smallest standard error, and no multicollinearity problem, it (Eq. 28) was selected to approximate the delay:

$$W_i = -0.035 + 0.506 \frac{\lambda_i(\sigma_{A_i}^{2} + \sigma_{S_i}^{2})(1 + C_{S_i}^{2})}{(1-\rho_i)(\frac{1}{\rho_i^2} + C_{S_i}^{2})} \tag{28}$$

(0.013) (0.001)

$R^2 = 0.9990$ $n = 120$ $s_e = 0.0151$ $\mu = 2.1602$

In this delay function, the average waiting time is almost proportional to the sum of the variances of interarrival times and service times. The average waiting time approaches infinity as the volume/capacity ratio ρ approaches 1.0.

Comparing Eqs. 27 and 28, it is seen that they are very similar. Theoretically, in Eq. 28, the parameter for second term should be equal to 0.5. Therefore, the first term should be negative since the second term is an upper bound for the average waiting time. Such comparison supports the reasonableness of the metamodel obtained in Eq. 28.

2.5 Algorithm For Two-Way Traffic Systems

The algorithm to estimate delays for a series of locks is developed based on the metamodels discussed in Section 2.1 to 2.4.

Delays depend on the arrival and service time distributions. Therefore, to estimate delays, we need to know in advance the means and variances of the interarrival and service time distributions. For two-way traffic systems with series of G/G/1 queues and bi-directional servers, a difficulty arises in identifying the variances of interarrival times. The variance of interarrival times at a certain lock labeled k is affected by the departure distributions from adjacent upstream and downstream locks. The departure distributions at adjacent upstream and downstream locks depend on their arrival distributions, which are affected by the departure distributions from Lock k. Hence, the variances of interarrival times at adjacent locks depend upon each other. Therefore, the variances of interarrival times cannot be determined from a single one-directional scan along a series of queues. For example, if we tried to estimate delays by scanning from upstream toward downstream, we could determine at Lock k the variance of interarrival times from upstream, but the variance of interarrival times from downstream would be unknown and would be affected by the departure distribution from this Lock k. To overcome such complex interdependence, an iterative algorithm is proposed. It starts scanning in one direction while using some initialized assumed values for the variances of interdeparture times from the opposite direction. It can thus sequentially estimate the interarrival and interdeparture time distributions for each lock. Then, the scanning direction is reversed and the process is repeated, using the interdeparture distributions for the opposite direction estimated in the previous scan. Alternating directions, the scanning process continues until the variances of interdeparture times computed in successive iterations converge. Then the algorithm stops reestimating the arrival distributions and proceeds to estimate delays.

In the first scan the initial values for variances of interdeparture times from the opposite direction are suggested to be equal or close to the variance at the corresponding locks from the scanning direction. Such assumed initial values hasten the converge but are not required. Brief convergence and stability tests, conducted by the authors, show that poor initial values increase the number of iterations for convergence but do not affect the final results. Computation speeds for various problem sizes and convergence criteria are summarized in Wei, Dai and Schonfeld [22].

The following algorithm is designed to apply the metamodels in estimating single-chamber lock delays. The notation used in this algorithm is previously discussed as follows:

$\bar{t}_{a_{j,i}}$: the average interarrival time for Direction j and Lock i

$\bar{t}_{d_{j,k}}$: the average interdeparture time for Direction j and Lock k

$\sigma_{a_{j,i_n}}^2$: variance of interarrival times for Direction j, Lock i and Iteration n

$\sigma_{d_{j,k_n}}^2$: variance of interdeparture times for Direction j, Lock k and Iteration n

$\sigma_{d_{1,O}}^2$: variance of interdeparture times from origin node

$\sigma_{d_{2,D}}^2$: variance of interdeparture times from destination node

$D_{i,k}$: distance between Locks i and k

$\mu_{v_{i,k}}$: average tow speed between Locks i and k

$\sigma_{v_{i,k}}$: standard deviation of tow speeds between Locks i and k

j : direction index (1 = downstream, 2 = upstream)

$\bar{t}_{A_i}$: the average interarrival time at Lock i

$\sigma_{A_{i_n}}^2$: variance of interarrival times at Lock i and Iteration n

$\bar{t}_{D_i}$: the average interdeparture time at Lock i

$\sigma_{D_{i_n}}^2$: variance of interdeparture times at Lock i and Iteration n

$\sigma_{S_i}^2$: variance of lock service times at Lock i

ρ_i : volume/capacity (=V/C) ratio at Lock i

λ_i : average inflow rate at Lock i

C_{S_i} : coefficient of variation of service time at Lock i

$C_{A_{i_n}}, C_{D_{i_n}}$: coefficients of variation for interarrival times and interdeparture times, respectively, at Lock i and Iteration n

c_i : constant (assumed value)

M : total number of Locks

W_i : the average waiting time at Lock i

The required input of this algorithm includes the service time distributions, inflow distributions, distances between locks, and speed distributions. This algorithm provides delays as well as interarrival and interdeparture time distributions. The algorithm consists of the following steps:

1. Compute the average directional interarrival times and the average directional interdeparture times for each lock.

$$\bar{t}_{d_{ji}} = \bar{t}_{a_{ji}} = \bar{t}_{d_{jk}} \quad \begin{cases} k=i-1, if\ j=1 \\ k=i+1, if\ j=2 \end{cases} \tag{1a}$$

2. Compute the average interarrival time for each lock

$$\bar{t}_{A_i} = \frac{\bar{t}_{a_{1,i}} * \bar{t}_{a_{2,i}}}{\bar{t}_{a_{1,i}} + \bar{t}_{a_{2,i}}} \quad i=1, \ldots M \tag{3a}$$

3. Estimate the variance of interarrival and interdeparture times for each lock.

3.1 Set n=1

3.2 Assume initial values for the variances of interdeparture times in Direction 2, at Locks 2 through M

$$\sigma_{d_{2,i_1}}{}^2 = c_p \quad i=2, \ldots M$$

3.3 Starting from Lock 1, let i = 1

3.4 Compute the variance of directional interarrival times at Lock i using the metamodel expressed in Eq. 2:

$$\sigma_{a_{ij_n}}{}^2 = -.354 + .998\sigma_{d_{kj_n}}{}^2 + .127\ln\frac{\sigma_{d_{kj_n}}{}^2 * D_{ik}}{\mu_{v_{ik}}} + .162\ln\sigma_{v_{ik}} \quad \begin{cases} k=i-1, if\ j=1 \\ k=i+1, if\ j=2 \end{cases} \tag{2a}$$

3.5 Compute the variance of combined interarrival times at Lock i

$$\sigma_{A_{i_n}}{}^2 = 0.0128 + 0.1243(\sigma_{a_{i1_n}}{}^2 + \sigma_{a_{i2_n}}{}^2) \tag{4a}$$

3.6 Compute the coefficient of variation for interarrival times at Lock i

$$C_{A_{i_n}} = \frac{\sigma_{A_{i_n}}}{\bar{t}_{A_i}}$$

3.7 Estimate the coefficient of variation for interdeparture times at Lock i using the metamodel expressed in Eq. 21:

$$C_{D_{i_n}}{}^2 = 0.396 + 0.606(C_{A_{i_n}}{}^2(1-\rho_i) + \rho_i) + C_{S_i}{}^2\rho_i^2 - \rho_i^2 \tag{21a}$$

3.8 Compute the variance of interdeparture times at Lock i

$$\sigma_{D_{i_n}}{}^2 = C_{D_{i_n}}{}^2 \bar{t}_{D_i}{}^2$$

3.9 Compute the variance of directional interdeparture times at Lock i

$$\sigma_{d_{ij_n}}{}^2 = 0.52 + 8.16\frac{\sigma_{a_{ij_n}}{}^2 * \sigma_{D_{i_n}}{}^2}{\sum_{j=1}^{2}\sigma_{a_{ij_n}}{}^2} \tag{26a}$$

3.10 Repeat Steps 3.5 - 3.9 for i = 2, ...M

3.11 Set n = n+1

3.12 Starting from Lock M, let i = M, and repeat Steps 3.5 - 3.10 for i = M, ...1

3.13 Set n = n+1

3.14 Repeat Steps 3.4 - 3.13

3.15 If the following condition is satisfied, then go to Step 4. Otherwise, go to Step 3.14

$$\frac{|\sigma_{A_{ij_n}}^2 - \sigma_{A_{ij_{n-1}}}^2|}{\sigma_{A_{ij_{n-1}}}^2} \leq 0.001 \quad i=1, \ldots M, \ j=1,2$$

4. Estimate the average waiting times using the metamodel expressed in Eq. 28:

$$W_i = -0.035 + 0.506 \frac{\lambda_i(\sigma_{A_{i_n}}^2 + \sigma_{S_i}^2)(1 + C_{S_i}^2)}{(1-\rho_i)(\frac{1}{\rho_i^2} + C_{S_i}^2)} \qquad \textbf{(28a)}$$

2.6 Algorithm For One-Way Traffic Systems

Although the numerical method was originally developed for two-way traffic systems, with a few simplifications, this method can be adapted for the more generally encountered systems with one-directional servers. One-directional systems may be treated as a special case of two-directional systems. The one-directional algorithm should perform better since the arrival distributions will be affected by the departure distributions from upstream only and are not subject to circular interdependence. Therefore, the arrival distributions may be determined in a single one-directional scan without any iteration. The notation used in this algorithm for one-way traffic is as follows:

D_{ik} : distance between Locks i and k

$\mu_{v_{ik}}$: average tow speed between Locks i and k

$\sigma_{v_{ik}}$: standard deviation of tow speeds between Locks i and k

$\bar{t}_{A_i}$: the average interarrival time at Lock i

$\sigma_{A_i}^2$: variance of interarrival times at Lock i

$\bar{t}_{D_i}$: average interdeparture time at Lock i

$\sigma_{D_i}^2$: variance of interdeparture times at Lock i

$\sigma_{A_0}^2$: variance of interdeparture times from origin node

$\sigma_{S_i}^2$: variance of lock service times at Lock i

ρ_i : V/C ratio at Lock i

λ_i : average inflow rate at Lock i

$C_{A_i}, C_{D_i}, C_{S_i}$: coefficients of variation for interarrival times, interdeparture times, and service times respectively, at Lock i.

M : total number of locks

W_i : the average waiting time at Lock i

This algorithm has the same input and output as the algorithm for two-way systems. It consists of the following steps:

1. Compute the average interarrival time and interdeparture time for each lock

$$\bar{t}_{D_i} = \bar{t}_{A_i} = \bar{t}_{D_{i-1}} \quad i=1, \ldots M \tag{1b}$$

2. Estimate the variance of interarrival time and interdeparture time for each lock.

2.1 Starting from Lock 1, let i=1

2.2 Compute the variance of interarrival times at Lock i using the metamodel developed for two-way traffic (Eq. 2):

$$\sigma_{A_i}^2 = -.354 + .998\sigma_{D_{i-1}}^2 + .127\ln\frac{\sigma_{D_{i-1}}^2 * D_{i,i-1}}{\mu_{v_{i,i-1}}} + .162\ln\sigma_{v_{i,i-1}} \tag{2b}$$

2.3 Compute the coefficient of variation for interarrival times at Lock i

$$C_{A_i} = \frac{\sigma_{A_i}}{\bar{t}_{A_i}}$$

2.4 Estimate the coefficient of variation for interdeparture times at Lock i using the metamodel developed for two-way traffic (Eq. 21):

$$C_{D_i}^2 = 0.396+0.606(C_{A_i}^2(1-\rho_i)+\rho_i)+C_{S_i}^2\rho_i^2-\rho_i^2 \qquad \text{(21b)}$$

2.5 Compute the variance of interdeparture times at Lock i

$$\sigma_{D_i}^2 = C_{D_i}^2 \bar{t}_{D_i}^2$$

2.6 Repeat steps 2.2 - 2.5 for i = 2, ...M

3. Estimate average waiting times using the metamodel developed for two-way traffic (Eq. 28):

$$W_i = -0.035+0.506\frac{\lambda_i(\sigma_{A_i}^2+\sigma_{S_i}^2)(1+C_{S_i}^2)}{(1-\rho_i)(\frac{1}{\rho_i^2}+C_{S_i}^2)} \qquad \text{(28b)}$$

2.7 Comparison of Numerical and Simulated Results

An experiment was conducted to test how well the numerical method duplicates the results of simulations. Three-lock, two-directional systems were tested. The controlled variables in this experiment include the V/C ratio ρ, the variance of lock service times, inflow rate, distance between locks, and tow speed. In this test the values of the controlled variables were uniformly sampling within the range of 0.01 to 0.89 for the V/C ratio, 0.0007 to 0.3332 for the variance of lock service times, 12.0 to 57.0 tows per day for the inflow rate, 5 to 60 miles for the distance between locks, 108 to 325 miles per day for the average tow speed, and 33.84 to 101.52 miles per day for the standard deviation of tow speeds. The ranges of the controlled variables were chosen to cover the possible operating conditions of waterways. For this system the variances of the interdeparture times converged within 0.1% for every lock and direction in no more than 5 iterations. The results are shown in Table 6.

The largest absolute deviation is 0.0944 hr when the average simulated waiting time is 2.3165 hr while the numerical method estimates 2.4109 hr. This represents a deviation of approximately 4.08%. The relative deviations are larger when V/C ratios and waiting times are close to zero, but very small in the more congested situations where investment evaluations have practical value. In general, these results indicate that the numerical method may be used to screen alternatives and greatly reduce the number of lock improvement combinations that have to be evaluated by the more detailed microscopic simulation model.

Table 6 Comparison of Numerical and Simulated Results (Two-Way Traffic System)

λ[1] tows/day	V/C	Dist mi	Speed mi/day	σ_s^2[2]	W_{sim}[3] hr	W_{num}[4] hr	Deviation hr
12.0	0.01	5	270	0.0007	0.0003	0.0000	-0.0003
12.0	0.07	5	270	0.0360	0.0153	0.0000	-0.0153
12.0	0.17	5	270	0.1897	0.0989	0.0636	-0.0353
24.0	0.15	5	325	0.0309	0.0334	0.0000	-0.0334
24.0	0.34	5	325	0.1620	0.2316	0.2029	-0.0287
24.0	0.25	5	325	0.0915	0.1139	0.0813	-0.0326
36.0	0.22	5	108	0.0309	0.0542	0.0198	-0.0344
36.0	0.03	5	108	0.0006	0.0008	0.0000	-0.0008
36.0	0.50	5	108	0.1618	0.4621	0.4588	-0.0033
48.0	0.50	5	162	0.1883	0.4355	0.4125	-0.0230
48.0	0.29	5	162	0.0646	0.0962	0.0698	-0.0264
48.0	0.67	5	162	0.3330	1.2028	1.1981	-0.0047
54.0	0.75	10	108	0.2271	1.3926	1.4037	0.0111
54.0	0.57	10	108	0.1279	0.3901	0.3782	-0.0119
54.0	0.89	10	108	0.3167	4.9837	4.9400	-0.0437
54.0	0.75	20	216	0.1616	1.2203	1.2222	0.0019
54.0	0.57	20	216	0.0909	0.0329	0.3076	-0.0210
54.0	0.89	20	216	0.2259	4.4608	4.4634	-0.0073
57.0	0.60	5	325	0.1557	0.5430	0.5579	0.0149
57.0	0.05	5	325	0.0011	0.0012	0.0000	-0.0012
57.0	0.80	5	325	0.2738	2.0874	2.1815	0.0941
57.0	0.35	60	162	0.0645	0.1372	0.1039	-0.0333
57.0	0.60	60	162	0.1882	0.6381	0.6465	0.0084
57.0	0.80	60	162	0.3322	2.3165	2.4109	0.0944

1 λ: two-way flow rate
2 σ_s^2: variance of service times
3 W_{sim}: simulated waiting times
4 W_{num}: waiting times estimated with numerical method

3. Conclusions and Recommendations

A numerical method has been developed for estimating delays through a series of queues with inflows and outflows occurring only at end nodes. This method was originally developed for systems with bi-directional servers. With a few simplifications, this method can be adapted for the more generally encountered systems with one-directional servers. The two-direction algorithm employs an iterative alternating direction scanning procedure to estimate the interarrival and interdeparture time distributions lock by lock until the interdeparture time variances for successive iterations converge. The performance of this two-direction algorithm is tested with satisfactory results. The one-direction algorithm only scans the interarrival time and interdeparture time distributions from the first to the last lock without any iteration and should, theoretically, be less subject to interdependence errors.

Both the two-direction and one-direction algorithms rely on several metamodels estimated from a previously developed simulation model. These metamodels provide the following valuable results for series of G/G/1 queues.

1. The delay function metamodel (Eq. 28) indicates how the V/C ratios, arrival distributions, and service distributions affect the average waiting times for G/G/1 queues. This delay function is inspired by the structure of Marchal's upper bound for G/G/1 queues. The standard error of the estimated delay is 0.0151 hours which, compared to its mean of 2.1602 hours, is sufficiently tight for evaluation purposes.
2. The relations among the coefficients of variation of interdeparture times, interarrival times, service times, and the V/C ratio are formulated in the departures metamodel. The structure of the departure function (Eq. 21) is based on functions for the squared coefficients of variation of interdeparture times. By applying Laplace transforms, these functions (i.e., Eqs. 16 and 17) are derived theoretically in this paper. Statistical estimation of the parameters yields a very good fit. The function's standard error of 0.000056 is extremely tight compared to its mean of 0.8267. The parameters also have very tight standard errors. In addition, this departure function is consistent with Burke's Theorem. The results show that the metamodelling approach combining queueing theory and statistical estimation based on simulation outputs is quite successful. This departure function should be very useful for analyzing networks of queues.

3. The arrivals module provides the relation between the variance of interarrival times and the variance of interdeparture times from the adjacent queue stations when speed variations change the headway distributions between successive queue stations.

The numerical method is useful in analyzing series of G/G/1 queues. However, the following extensions would be desirable to increase its applicability.

1. A function should be developed to estimate the variance of interarrival times when inflows and outflows occur between queue stations. With such an arrival function the numerical method might be applied to general networks of queues, including tree and grid networks. The proposed approach would compute the variance of overall arrival rates as the sum of the arrival rate variances from all inflows, assuming individual inflows are independent of each other, and then develop the relation between the variance of interarrival times and the variance of arrival rates.
2. The effects of random failures (i.e., stalls) might be incorporated by treating stalls as a second class of users with its own arrival and service time distributions.
3. If possible, the numerical method should be extended to locks with two or more chambers. This is rather difficult for chambers with different characteristics, because the chamber assignment process affects lock capacity.
4. Queues with limited storage.

Additional statistical and computational tests are also desirable to further validate this method and extend its applicability.

The final methodology for estimating waterway delays may combine simulation and numerical methods. The simulation model may be used to accurately estimate delays for relatively small systems of locks. The numerical method may be used in analyzing large combinatorial problems. Guidelines should be developed for switching between the numerical method and simulation in applications requiring intermediate accuracy.

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